The Correlation Between Prices and Output:
Controlling for Contaminating Dynamics

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Sharp attention has recently been re-focused on the correlation between the price level and aggregate output. For many years this issue appeared settled. The Phillips curve portrayed a negative short-run relationship between the inflation and unemployment rates and so indicated that prices were generally procyclical. Lucas (1977), following Burns and Mitchell (1946), took procyclical prices as one of the facts that models of the business cycle should explain. This "fact" was challenged by Kydland and Prescott (1990) and Cooley and Ohanian (1991). These authors argued that for the post-Korean War era the price level has been countercyclical. Wolf (1990) suggested that the negative correlation was primarily the result of the oil price shocks in the 1970s. Ravn and Sola (1995) study this question with a version of Hamilton’s switching regression methodology to investigate shifts in the correlation between prices and output. They conclude that in general the negative correlation is robust to changes in regimes, including regimes that are not characterized by oil price shocks.

Initially, empirical evidence on the correlation between prices and output was taken as testimony on the veracity of economic models. The positive correlation was taken as evidence that demand shocks were most important for short-run economic behavior, and when the evidence mounted that the correlation was negative it was taken as evidence of supply shocks driving the economy. Hall (1995) and Judd and Trehan (1995) questioned the use of the price-output correlation in weighing the relative merits of models. They argued that dynamic movements of prices and output may mask the true nature of the shock. For example, in a textbook AD-AS model with a flat short-run supply curve, an increase in aggregate demand increases output but not the price level. However, as prices adjust upward, output will fall back toward its long run equilibrium level. Because of this dynamic adjustment, the data would show a negative correlation between prices and output even though the shock originated on the demand side. Thus, an economy driven solely by demand shocks may well produce a negative price-output correlation.

In this paper we purge the dynamic effects from the correlations between price and output by examining the residuals from a dynamic empirical model. This strategy is straightforward. We first estimate reduced form price and output equations. These equations control for the dynamic effects emphasized by Hall, and Judd and Tehran. The residuals from these equations contain the effects of both demand and supply shocks. The correlation between these two residuals provides
evidence on the relative strengths of the contemporaneous demand and supply shocks. Our strategy may be viewed as a generalization of "detrending". Typically, variables are detrended by first differencing or by applying a Hodrick-Prescott filter. However, these simple methods fail to remove the short-run dynamic interrelationships between the variables.

Our approach most closely follows the methodology of den Haan (1996). He estimates a VAR to control for dynamic effects, and examines the correlations between the forecast errors for prices and output at different horizons. He finds, like us, that the one-period ahead errors have a small positive correlation. At longer horizons he finds that the errors are negatively correlated. He interprets these results as consistent with demand shocks being the most important shocks for short-run fluctuations, but supply shocks dominating in the long run.1 Our results differ from den Haan’s in two important ways. First, we also examine the correlations for Canada and the United Kingdom, while den Haan uses only U.S. data. In both of these countries, and in contrast to the United States, the short run correlations are negative. Second, we study the period-by-period errors in all three countries and find for each country that both supply and demand shocks are significant contributors to short-run fluctuations.

The remainder of this paper is organized as follows. In the next section we describe our methodology. In section 3 we discuss the specification of the model and we present our results in section 4. We conclude in section 5.

2. The Method

We can describe the economy by specifying the endogenous variables, the exogenous variables, the contemporaneous demand and supply shocks, \( e^d_t \) and \( e^s_t \) respectively, and the relationships between the variables and the shocks. We assume that the shocks are independent white noise, but otherwise specify things in a very general way. Let \( Y_t \) be the level of real output, \( P_t \) be the price level, \( Z_t \) be another endogenous variable, which may be interpreted as a vector, and \( X_t \) be an exogenous variable, which may also have a vector interpretation. The

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1 He also examines the relationship between real wages and output.
economy is defined by the system of implicit equations:

\[ F^1(Y_t, P_t, Z_t, X_t, Y_{t-1}, P_{t-1}, Z_{t-1}, X_{t-1}, \ldots, e_t^d, e_t^s) = 0 \]

\[ F^2(Y_t, P_t, Z_t, X_t, Y_{t-1}, P_{t-1}, Z_{t-1}, X_{t-1}, \ldots, e_t^d, e_t^s) = 0 \]

\[ F^3(Y_t, P_t, Z_t, X_t, Y_{t-1}, P_{t-1}, Z_{t-1}, X_{t-1}, \ldots, e_t^d, e_t^s) = 0 \]

The lagged variables are included to capture any endogenous dynamics in the model and the information value of these variables in the formation of expectations. Note that if \( Z \) is given a vector interpretation, the function \( F^3 \) must also be given a vector interpretation.

We now think of solving this system for the equilibrium values of the endogenous variables and then linearizing the results. Part of the solution will be in the form

\[ Y_t = A(L)Y_t + B(L)P_t + C(L)Z_t + D(L)X_t + u_t^y \] (1)

\[ P_t = \Psi(L)Y_t + \Omega(L)P_t + \Lambda(L)Z_t + \Gamma(L)X_t + u_t^p \] (2)

where \( A(L) \) and the like are polynomial lag operators, and the shocks are given by

\[ u_t^y = a e_t^d + b e_t^s \]

\[ u_t^p = \alpha e_t^d + \beta e_t^s \]

If we interpret equations (1) and (2) as forecasting equations, then these terms are the forecast errors from the output and price equations. The variances of these errors or shocks are

\[ \sigma_y^2 = a^2 \sigma_d^2 + b^2 \sigma_s^2 \]

\[ \sigma_p^2 = \alpha^2 \sigma_d^2 + \beta^2 \sigma_s^2 \]
The structure of this solution shows the problem identified by Hall, and Judd and Trehan. To see the nature of this problem, ignore detrending issues, assume each variable has zero mean, and suppose that only one lagged value of $Y_t$, $P_t$, and $Z_t$ enter equations (1) and (2). In this case the covariance between $Y_t$ and $P_t$ is given by

$$E_{Y_tP_t} = E(aY_{t-1} + bP_{t-1} + cZ_{t-1} + u_t^Y)(\psi Y_{t-1} + \omega P_{t-1} + \lambda Z_{t-1} + u_t^P)$$

where $\theta_1 = k \cdot (a\psi)$, $\theta_2 = k \cdot (b\omega)$, $\theta_3 = k \cdot (c\lambda)$, $\theta_4 = k \cdot (b\lambda + c\omega)$, $\theta_5 = k \cdot (a\lambda + c\psi)$, and $k = 1/(1 - a\omega - b\psi)$. First, note that if there are no dynamics, the coefficients on the lagged variables are all zero, so the $\theta$'s are all zero, and the covariance of $Y$ and $P$ is just the covariance of the shocks $u_P$ and $u_Y$ in the two equations. However, once dynamics are allowed, we see from equation (3) that the covariance of $Y$ and $P$ depends not only the covariance of the shocks $u_P$ and $u_Y$, but also on the variances of $Y$, $P$, and $Z$; and the covariances of $Y$ and $Z$, and $P$ and $Z$; and the values of the coefficients. Detrending methods that render $Y$ and $P$ stationary do not fully purge the contaminating dynamics. For example, first-differencing would mean the variances and covariances in equation (3) are variances and covariances of first differences rather than levels. It does not make the covariances zero. We purge the dynamics by controlling for the dynamic elements and estimating $u_P$ and $u_Y$ directly.

To estimate $u_P$ and $u_Y$ we must first decide whether to estimate the model in levels or in first-differences. A closely related question was recently raised by Chada and Prasad (1994). They wondered whether it was appropriate to examine the correlation between output and the price level or the correlation between output and inflation. This question is non-trivial since with their data the sign of the correlation depends on the answer. There does not appear to be a clear theoretical reason to prefer one correlation over the other, so we, in effect, let the data decide. We estimate equations (1) and (2) in levels. The OLS estimates of the coefficients will be consistent and so yield consistent estimates of the residuals.²

The specification in levels is sufficiently flexible to allow inflation or the price level to be the relevant variable. For example, equation (1) encompasses, among others, the following two possibilities:

\[ \Delta Y_t = a_1 \Delta P_{t-1} + a_2 \Delta P_{t-2} + \cdots \]

\[ \Delta Y_t = a_1 \Delta \Delta P_{t-1} + a_2 \Delta \Delta P_{t-2} + \cdots \]

The first case suggests that the relevant relationship is between output growth and inflation, while in the second case the relevant relationship appears to be between output growth and the changes in the inflation rate.

We estimate equations (1) and (2) and capture the estimates of the residuals \( u^Y \) and \( u^P \). The expected value of the product of the residuals is

\[ E[u^Y \cdot u^P] = \alpha a \sigma_d^2 + \beta b \sigma_s^2 \]

Positive demand shocks raise both prices and output in the current period, so we expect \( a \) and \( \alpha \) to be positive. On the other hand, positive supply shocks raise output, but cause prices to fall; so we take \( b \) to be positive, but \( \beta \) to be negative. Thus, the coefficient on the variance of supply shocks should be negative, while the coefficient on the variance of the demand shock should be positive. The role of demand and supply shocks in determining the size of the variance depends on two separate effects. First, the relative size of the variances of the shocks is important. For example, if variations in aggregate demand are large relative to variations in aggregate supply, the product of the residuals will tend to be positive. Second, the size of the coefficients on the variance terms reflect the relative impact effect of the shocks. It is possible that the variances of the shocks are the same, but the impact effect of, say, supply shocks is far larger. In this case the product will tend to be negative. Unfortunately, there is no way to disentangle the variance and impact effects.

The sample covariance or correlation coefficient is an estimate of the average nature of the shocks. Additional information can be gathered by examining period-by-period shocks. This information permits an informal check on the robustness of the results since at least some of the
shocks should correspond to observable historical events. We look at both the sample correlation and the period-by-period cross-products of the price and output residuals for the United States, Canada, and the United Kingdom.

3. Specification

The variables included for the United States specification are the natural logs of real GDP, the GDP deflator, the relative price of oil, M1, the S&P 500, the levels of the federal funds rate, the ratio of net exports to GDP, the spread between the 3 month treasury bill rate and 10 year treasury bond rate, and the ratio of defense spending to GDP. For the U.S. consistent measures of the money supply begin in 1959 and so does our sample. The import and export data and the rates on treasury securities comes from International Financial Statistics (on CD-rom). The remainder comes from Citibase.

The relative price of oil is included because several economists have focused on its role in recessions (see Hamilton (1983) for example). The federal funds rate and M1 are often indicators of monetary and credit policy. The S&P 500 is included to proxy for changes in the productivity of capital (see, for example, Barro and Sala-i-Martin (1990)). The ratio of defense spending to GDP is a measure of fiscal policy. The spread between short and long term rates of interest proxies for conditions in credit markets and the ratio of net exports to GDP proxies for disturbances to the international sector. In general, we thought it was better to begin with a generous specification since omitting a relevant variable would fail to purge the dynamics. We find below that the results are similar for a more parsimonious specification.

For Canada and the United Kingdom our data comes exclusively from International Financial Statistics and our sample begins in 1957. The variables included for Canada and the U.K. are the natural logs of real GDP, the GDP deflator, narrow money, the price of oil, the levels of the interest rate on 3 month government bills, the spread between long and short term government securities, the ratio of government consumption to GDP, and the ratio of net exports to GDP. To generate the price of oil we converted the U.S. price of oil into Canadian or British currency. All of the data is quarterly and the general specification for each country includes eight lags of each variables in both the price and output equations.
4. Results

a. the United States

Once we obtained the residuals from the estimation of equations (1) and (2), we tested for a unit root in the residuals with an augmented Dickey-Fuller test. For both equations, the hypothesis of a unit root was soundly rejected. The Breusch-Pagan LM test for serial correlation failed to reject the null of no first-order serial correlation at the 10% level for both equations. We also investigated the CUSUM and CUSUMSQ plots to examine the stability of the model. The CUSUMSQ does not cross the bounds of the 5% lines for either equation. With the CUSUM the price equation shows some evidence of instability in the mid-1980s.

The sample correlation coefficient between the forecast errors in the price and output equations for the U.S. is positive, but relatively small at .12. This number may occur for two different reasons. First, it could be the case that most of the shocks to the economy are small demand shocks. An alternative possibility is that the economy is buffeted by about equal quantities of demand and supply shocks.

Figure 1 plots the quarter-by-quarter cross-product of the residual estimates from the two equations. The shaded areas are recessions. If a prevalence of small demand shocks is responsible for the correlation, then most of the cross-products should be positive. On the other hand, if both types of shocks disturb the economy, then we should observe at least some large positive cross-products. There are three instances of very large cross-products: one in 1960, another in 1970, and a third in 1978. Two of these are positive and one is negative. The first two are associated with recessions. The third is the result of the Blizzard of 1978 and a concurrent sharp increase in the price of food. The 1979 Economic Report of the President assigns the extraordinary growth in the second quarter of 1978 to the recovery from the blizzard, which was over 12% and is the largest for any quarter in sample. In the same quarter inflation was over 10%, the second highest rate in our sample. Much of this price rise was fueled by a 20% increase in the price of food. This observation is not very influential on the pattern of the cross-products.

3 The CUSUM AND CUSUMSQ plots are generated by recursively estimating the model, capturing the residuals, and plotting them. In the CUSUM case the cumulative sum of the residuals is plotted whereas for CUSUMSQ the cumulative sum of the squared residuals are plotted. The two plots are complementary methods of examining a model's stability. See Harvey (1991) pp. 153-55.
Figure 2 plots these cross-products when a dummy variable is included for the 1978.2 and the movements, with the exception of the 1978 episode, are very close to those in Figure 1. However, this observation does contribute significantly to the correlation over the entire sample. When a dummy for 1978 is included, the sample correlation falls to .04.

If we define large by greater than one standard deviation, then there are twelve large demand and ten large supply shocks. The mid-1960s appear to be dominated by variations in demand. The correlation coefficient from the first quarter of 1964 to the last quarter of 1968 is .21. On the other hand, the 1970s appear to be dominated by supply shocks. From 1970:1 to 1977:4 the correlation coefficient is -.25 and the OPEC recession of 1974 is associated with large supply shocks. The recession in 1970 also appears to be associated with a large supply shock, while large positive cross-products accompany the 1980 and 1960 recessions. The 1982 and 1990 recessions do not appear driven by a single type of shock.

In sum, the examination of the cross-products of the residuals indicates that there have been substantial shocks of both types in about equal frequency, and this accounts for the relatively

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small sample correlation coefficient.

To check the sensitivity of our results to the variables included in the model we estimated the price and output equations with only lagged values of prices and output as explanatory variables. The residuals from both equations are stationary. For this specification there was some evidence of first-order serial correlation. The Breusch-Pagan LM test rejects first-order correlation at the 5% level the output equation, but not the price equation. The sample correlation between the residuals from the two equations is .10, very close to the figure from the general specification. The correlation between the cross-products from the two specifications is .60 and the general pattern of the cross-products is very similar as can be seen from Figure 2. In short, the conclusions are robust to these changes in specification.

b. Canada and the United Kingdom

We now turn to Canada and the United Kingdom, and we take up the U.K. first. We estimated the reduced form equations (1) and (2) and subjected the residuals to the same tests as the residuals for the U.S. We found that the residuals for both the price and output equations are stationary. We found evidence of first-order serial correlation in the price equation for the U.K. However, when we adjusted for serial correlation our conclusions were unaffected. As a result, the figures and discussion of the U.K case are for the uncorrected price equation. Neither the CUSUM or the CUSUMSQ test revealed any instability.
The sample correlation coefficient for the U.K. is -.15. In contrast to the U.S., the correlation is negative; but, like the United States, it is relatively small. The cross-product of the residuals are plotted in Figure 3. We see both large positive and negative cross-products suggesting that the U.K. economy, like the U.S., is subject to significant demand and supply shocks. The 1970s are again a time of apparent supply shocks with the correlation coefficient over this decade equal to -.23, and the protracted recession in the early 1980s is not accompanied by a large cross-product.\(^5\)

We again checked for the sensitivity of our specification by estimating the price and output equations with only lagged values of prices and output. The residuals from both equations are stationary and there is no evidence of first-order serial correlation in either equation. For this case the sample correlation between the cross-products is -.22. The correlation of the cross-products across the specifications is .61 and again the pattern is similar in both cases.

The residuals for Canada are stationary, reject the presence of first-order serial correlation, and are stable. The sample correlation coefficient of the residuals from the reduced form

\(^5\) We define the peak of an expansion as the quarter that began at least two consecutive quarters of negative growth. We define a trough in the quarter in which two consecutive quarters of positive growth began.
equations is -.30. The plot of the cross-products of the residuals is shown in Figure 4. There are fewer large shocks for Canada than for the U.S. and U.K. These large shocks are roughly equally divided between supply and demand shocks, so the story seems to be the same as for the U.S. and the U.K. A sensitivity check like those above yields a sample correlation between cross-products of -.33 and a correlation across specifications of .54.

5. Conclusion

The correlation between price and output has been used to judge the source of shocks. Hall, and Judd and Trehan questioned this interpretation because the dynamic adjustment of prices and output mask the origin of a shock. We have purged the correlation of these dynamic effects by estimating reduced form equations for prices and output. With our approach the correlations are typically small, ranging from .12 in the U.S. to -.33 for the sparse specification for Canada. By examining the cross-product of the residuals from the price and output equations we get a view of the nature of the quarter-by-quarter shocks. An inspection of these cross-products
suggests that shocks come from both sides in about equal proportions. Nevertheless, some periods happen to be dominated by supply shocks while others are driven primarily by demand shocks, and recessions can be generated by a large shock of either type or a combination of both demand and supply shocks.

In a recent paper Cochrane (1994) found that none of the usual suspects, money, technology, oil shocks, or credit shocks can robustly account for the bulk of economic fluctuations. If none of them are individually guilty, perhaps they are collectively guilty. It may be the case that several different types of shocks, some easily observable and some not, conspire to produce the business cycle.

References


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Judd, John P. and Bharat Trehan "The Cyclic Behavior of Prices" *Journal of Money, Credit, and Banking*, August 1995, 789-797.


### Summary Table

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**United Kingdom**

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notes: The entry in the lm column are p-values. The dickey-fuller stats are 't' stats. Stable means that the plot of CUSUM and CUSUMSQ stay within the 5% bounds.