IS TAX SHARING OPTIMAL?
AN ANALYSIS IN A PRINCIPAL-AGENT FRAMEWORK

BARNALI GUPTA1 AND CHRISTELLE VIAUROUX2

ABSTRACT. We study the effects of a statutory wage tax sharing rule in a principal-agent framework with moral hazard (à la Holmstrom, 1979.) The analysis indicates that tax sharing with positive legislated contributions from both the employer and employee does not maximize any of the relevant outcomes – employee effort, wages, profits or welfare. Moreover, a rule which specifies a corner solution, with 100% of the tax statutorily levied on the employer will maximize expected profit and expected welfare while 100% of the tax statutorily levied on the employee will maximize effort and expected wages. This indicates that the neutrality result vis-a-vis tax sharing rules, that applies to standard complete information competitive market models cannot be widely generalized.

JEL codes: D8, H2

Keywords: moral hazard, taxes

1. INTRODUCTION

During the past three decades, the principal-agent framework has become an integral part of economic modeling. (See Sappington, 1991, and Laffont and Martimort, 2002). When the principal cannot observe the agent’s effort, she designs an optimal compensation schedule to induce optimal effort. Frequently, policies enforced by a third party have substantial impact on the optimal contract. The common practice of wage taxation is one such policy.

The taxation of wage income in various forms, is common throughout the world. In approximately half of all OECD countries, the shares of employer/employee contributions toward a social security tax, for example, have been stable at approximately 25% of total labor costs. Yet, the distribution of this share between employer/employee varies across countries. There is a 50:50 split in Germany, Switzerland, United States, Luxembourg and Japan. In most other countries, employers typically pay the major share. The exceptions are Denmark and the Netherlands, where employees generally pay the most. This variation and the lack of formal analysis in the literature, motivate the present study.

The question investigated in this paper is the effect of a statutory wage tax sharing rule on wages, effort, profits and aggregate welfare. In a typical complete information competitive model, the standard result of the neutrality of tax shares is well known. However, incomplete information and imperfect competition are more often the rule, rather than exceptions. In that context, the effect of tax sharing rules and any analysis on the optimality of these rules, is not especially evident in the literature. We show that the same neutrality
result does not apply in this framework. Our model is a principal (employer) - agent (employee) framework with moral hazard (à la Holmstrom, 1979) where an employee’s effort is unobservable by his employer and at the same time, is affected by the taxes imposed on his wage income. The tax sharing rule is set ex ante by the government—prior to any decisions made by either the principal or the agent.

Our results show that tax sharing with positive legislated contributions from both the employer and employee, does not maximize any of the relevant outcomes: effort, wages, profits or welfare. Moreover, it turns out that a statutory sharing rule which specifies that 100% of the tax statutorily levied on the employer will, under some conditions, maximize expected profit and welfare while 100% of the tax statutorily levied on the employee will, under some conditions, maximize effort and expected wages.

There is a substantial literature on the optimal income tax in adverse selection models (see Diamond, 1998 and Seade, 1977). The focus of our work is on moral hazard and risk sharing. In the theoretical literature studying the impact of taxes on hours of work, the typical conflict between the substitution effect and income effect has rendered any conclusion logically indeterminate. While this paper is related to the topic of taxation under uncertainty (see Eaton and Rosen 1980 a, b; Rosen, 1980), it is fundamentally different in the model being used and the implication of a statutory tax sharing rule that is being studied. It is useful to see the work by Feldstein (1995) for discussion on the extent to which taxable income as a whole, and not just labor supply, responds to changes in marginal tax rates. While not directly related to our work, Banerjee and Besley (1990) study optimal tax intervention with moral hazard, when insurance markets are incomplete.

The purpose of this work is to make the case that the standard neutrality result vis-a-vis tax sharing rule, does not necessarily extend beyond the complete information, competitive market paradigm. Moreover, positive tax shares for both employer and employee, do not maximize any of the relevant outcomes. The next section describes the model. Section 3 presents the analysis while Section 4 concludes and discusses some extensions.

2. Model

The basic framework is the familiar one of a risk neutral employer (or principal) and risk averse employee (or agent) who works for wages, which are taxed by the government. Further, a statutory distribution of the tax between the employer and employee, is mandated by the government. The employer-employee relationship involves moral hazard, where the agent’s effort is unobservable by the principal, but it affects the expected outcome as well as the riskiness of outcomes. The realized output is a noisy signal of the agent’s effort. Therefore the principal wants to use the contract to induce the agent to exert optimal effort.

Let:
- \( a \): agent’s effort
- \( x \in [0, \infty) \): output
- \( f(x|a) \): conditional density function
- \( w(x) \): wage payment from the principal to the agent when output \( x \) is realized
- \( U_A(w, a) \): agent’s utility function, where \( U_A(w, a) = 2(w)^\eta - a^2 \). We set \( \eta = 1/2 \) to satisfy Jewitt (1988) conditions, so the first order approach can be used.\(^2\)

\(^1\)Eaton and Rosen (1980 a) summarize the extensive econometric research as suggestive of very small responses in hours of work to changes in net wage for prime male earners. However, other groups, such as married women, have considerably higher labor supply response rates.

\(^2\)Note that \( \eta \leq \frac{1}{2} \), will satisfy Jewitt’s conditions. We use \( \eta = 1/2 \), to get explicit solutions.
$U$: agent’s reservation utility

$U^P(x, w) = \pi(x, w)$: principal’s payoff function, where $U^P(x, w) = x - w$.

$t$: tax rate on wage $w(x)$, set by the government

$\gamma$: employee’s share of wage tax.

$W(x)$: welfare, where $W(x) = x - (1 - t\gamma)w(x) + 2(1 - t\gamma)^{1/2}w(x)^{1/2} - a^2$.

Note that we use the gamma distribution, because its flexibility and general properties are well suited for use in this framework. See Bose, Pal and Sappington, (2007) for this idea and justification for using this distribution in a principal-agent framework. The density function for the gamma distribution is given by:

$$f(x|a) = \frac{1}{\Gamma(p)} x^{(p)-1} e^{-x/a}, \text{ for } x \in [0, \infty)$$

where

$$\Gamma(p) = \int_0^\infty e^{-x} x^{(p)-1} dx.$$

3. Results

We use the standard principal agent framework with moral hazard. The Principal’s problem $[P]$ can be written as:

Maximize$_{w, a} \int_0^\infty [x - w(x) - (1 - \gamma)tw(x)] f(x|a) dx$

subject to the Participation Constraint

\begin{equation}
\int_0^\infty 2\sqrt{w(x) - t\gamma w(x)} f(x|a) dx - a^2 = U,
\end{equation}

and the Incentive Compatibility Constraint

\begin{equation}
\int_0^\infty 2\sqrt{w(x) - t\gamma w(x)} f_a(x|a) dx - 2a = 0.
\end{equation}

The solution to $[P]$ is characterized in Theorem 1. We assume that $U$ is sufficiently large such that $w'(x) > 0$ for all $x \geq 0$.

**Theorem 1.** The solution to the Principal’s Problem $[P]$ (second best solution) is characterized by the following set of equations:

\begin{align}
\lambda &= \frac{1 + t - t\gamma}{2(1 - t\gamma)} (a^2 + U), \\
\mu &= \frac{(1 + t - t\gamma)a^3}{p(1 - t\gamma)}, \\
\alpha &= \sqrt[3]{-L + \sqrt{L^2 + K^3}} + \sqrt[3]{-L - \sqrt{L^2 + K^3}}, \\
w(x) &= \frac{1}{4(1 - t\gamma)} \left( \frac{2a}{p} x + U - a^2 \right)^2, \\
K &= \frac{pU}{3(p + 4)} \quad \text{and} \quad L := -\frac{p^2(1 - t\gamma)}{2(p + 4)(1 + t - t\gamma)}.
\end{align}

**Proof.** See Appendix 1.
Given the solution to \([P]\), we are next interested in the relationship between the employee’s tax share \(\gamma\) and his optimal effort \(a\), expected wage \(E(w)\), the principal’s expected profit \(E(\pi)\) and the expected aggregate welfare, \(E(W)\). These results are presented in Theorem 2.

**Theorem 2.** The following relations hold for all tax shares \(\gamma \in [0, 1]\):

(a) \(\frac{\partial a}{\partial \gamma} < 0\);

(b) If \(1 - t > (2 + \gamma)t\), then \(\frac{\partial E(w)}{\partial \gamma} > 0\);

(c) \(\frac{\partial E(\pi)}{\partial \gamma} > 0\);

(d) \(\frac{\partial E(W)}{\partial \gamma} < 0\).

where \(E(w)\) is expected wage, \(E(\pi)\) is expected profit and \(E(W)\) is expected welfare.

**Proof.** See Appendix 2. \(\square\)

From Theorem 2(a), we see that the agent’s optimal effort is a strictly declining function of his share of the wage tax. The higher his mandated taxed share of wages, the lower his effort. Since the agent is risk averse, ceteris paribus, he is less likely to put in effort as his expected post tax income falls.

If the share of net wages \((1 - t)\), is at least three times the taxed share \(t\), then the expected wage increases with the share of the tax paid by the employee, \(\gamma\), as stated in Theorem 2(b). Computation results in the next section show that this result is quite robust. The implication is that in the face of uncertainty, the tax share matters because it affects the ability of the principal to trade-off risk sharing versus incentives.

For this same reason, the principal’s expected profit falls with \(\gamma\), as stated in Theorem 2(c). The constraint on the principal’s ability to trade-off risk sharing versus incentives, lowers her expected profit.

It turns out, from 2(d), that expected welfare is a strictly decreasing function of the agent’s share of the wage tax. These results imply that while expected wage is maximized if the agent is legislatively mandated to pay 100% of the wage tax, the agent’s effort, expected profit and in aggregate, expected welfare, are all minimized under that tax rule. What is abundantly clear is that regardless of the tax distribution chosen as a policy matter, an interior solution with positive shares for both principal and agent, does not optimize any of these outcomes. From this perspective, it is difficult to justify an interior distribution of tax shares.

We explore further, the theoretical findings in Theorem 2 with numerical computation to check the robustness of the conclusions.

### 3.1. Computation results.

The numerical results largely verify the robustness of the solutions from Theorem 2.

**Conclusion 1.** Employee effort \(a\), expected firm profit \(E(\pi)\) and expected welfare \(E(W)\) are maximized when the statutorily mandated employee’s share of the tax \(\gamma\) is zero; employee expected wage \(E(w)\) is maximized when the statutorily mandated employee’s share of the tax is one.
Tables 1-4 report the numerical results for $p = 3$, $U_0 = 1.5$, $t$ and $\gamma$ varying from 0.1 to 0.9 in increments of 0.1. The results corresponding to values of $t$, $\gamma$, $p$ and $U_0$ satisfying the sufficient conditions in Theorem 2 are highlighted in bold. Consistent with Theorem 2(a), given $t$, employee effort decreases with an increase in $\gamma$ (see Table 1). We also see that given $t$, the expected wage increases with $\gamma$. Note that this positive monotonic relationship is actually more robust than the sufficient condition in Theorem 2(b) might indicate (see Table 2). As stated in Theorem 2(c), computation results verify that expected profit decreases with $\gamma$ for given $t$ (see Table 3). Simulation results in Table 4 support the result in Theorem 2(d), that expected welfare falls with $\gamma$, given $t$.

4. Conclusion

In the presence of incomplete information, the statutory liability of a tax has very clear implications for profits, wages and aggregate welfare. The theoretical and numerical results do not find any justification for distributing the burden of a wage tax between employer and employee. While the results are derived using specific functions, the point we wish to make is quite general—the conclusions about the effects of statutory tax liability from the complete information competitive market framework, cannot be carried forward without further study. Clearly the moral hazard intrinsic in the second best case is critically important to the results obtained here. In the first best case, it can be shown relatively easily that profit and welfare are maximized when the employee’s statutory tax share is 100%, while simultaneously wages and tax revenue are minimized.

In an extension of this analysis, we relax the assumption of fixed reservation utility and allow the agent’s opportunity wage to depend on the tax environment. Allowing $U = U_0(1 - t\gamma)^\theta$, $(\theta > 0)$ such that $\theta$ is the elasticity of the agent’s utility with respect to post tax share of wage, we find sufficient conditions on $\theta$ such that as long as the agent’s reservation utility is not "too responsive" to changes in the share of wages that must be paid in taxes, the results from Theorem 2, with fixed reservation utility, are generally robust. The conclusions suggest a careful examination that explores the connection between mandated tax liability, its implications for employer and employee earnings and what the optimal policy should be in this context, is warranted.

References

Recall the following: The gamma distribution has many general properties that are well suited for use in this framework. We recall in particular that

\[ (Ax+B)^2 f(x|a) \, dx = A^2 a^2 p(p+1) + 2ABp + B^2 = A^2 a^2 p + (Aa + B)^2 \]

for any constants \( A \) and \( B \) and that

\[ f_a(x|a) = \frac{x - ap}{a^2} \]

where \( f_a \) is the partial derivative of \( f(x|a) \).

Write

\[ \alpha := 1 + (1 - \gamma)t \quad \text{and} \quad \beta := 2\sqrt{(1 - t\gamma)} \]

for brevity. Then the Principal’s Problem \([P]\) can be re-written as follows:

Maximize \( w, a \int_0^\infty \left[ x - \alpha w(x) \right] f(x|a) \, dx \)

subject to

\[ \int_0^\infty \beta \sqrt{w(x)} f(x|a) \, dx - a^2 = \bar{U} \]

and

\[ \int_0^\infty \beta \sqrt{w(x)} f_a(x|a) \, dx - 2a = 0. \]

The Lagrangian corresponding to the Principal’s Problem is:

\[ \mathcal{L} = \int_0^\infty \left[ x - \alpha w(x) \right] f(x|a) \, dx + \lambda \left[ \int_0^\infty \beta \sqrt{w(x)} f(x|a) \, dx - a^2 - \bar{U} \right] + \mu \left[ \int_0^\infty \beta \sqrt{w(x)} f_a(x|a) \, dx - 2a \right]. \]

Theorem 1 follows from the maximization of \( \mathcal{L} \) with respect to \( w, \lambda, \mu \) and \( a \).

Step 1. Pointwise maximization of \( \mathcal{L} \) with respect to \( w \) yields

\[ \frac{\partial \mathcal{L}}{\partial w} = -\alpha f(x|a) + \frac{\lambda \beta}{2 \sqrt{w(x)}} f(x|a) + \frac{\mu \beta}{2 \sqrt{w(x)}} f_a(x|a) = 0 \]

Solving for \( w(x) \) using (5.2) gives

\[ w(x) = \left[ \frac{\beta}{2\alpha} \left\{ \lambda + \frac{x - ap}{a^2} \right\} \right]^2. \]
Step 2. Maximization of $\mathcal{L}$ with respect to $\lambda$, using (5.4) gives:

$$
\int_0^{\infty} \beta \sqrt{w(x)} f(x|\alpha) dx - a^2 - U = 0
$$

$$
\frac{\beta^2}{2\alpha} \int_0^{\infty} \left[ \lambda + \frac{x - a p}{a^2} \right] f(x|\alpha) dx = a^2 + U
$$

Noting that

(5.5) \(\int_0^{\infty} \frac{x - a p}{a^2} f(x|\alpha) dx = 0\)

we have:

(5.6) \(\lambda = \frac{2\alpha}{\beta^2} (a^2 + U)\).

where the last equality follows from (5.1).

Step 3. Maximization of $\mathcal{L}$ with respect to $\mu$ yields:

(5.7) \(\int_0^{\infty} \beta \sqrt{w(x)} f_a(x|\alpha) dx - 2a = 0\).

Using (5.2), (5.4) and (5.6), we have

$$
\frac{\mu \beta^2}{2\alpha a^4} \int_0^{\infty} (x - a p)^2 f(x|\alpha) dx - 2a = 0,
$$

which gives using (5.1) and solving for $\mu$

$$
\mu = \frac{4\alpha a^3}{\beta^2 p}.
$$

Finally, plugging the expression of $\mu$, $\lambda$, $\alpha$ and $\beta$ above gives the expression of the wage function (3.6).

Step 4. Maximization of $\mathcal{L}$ with respect to $\alpha$ yields:

(5.8) \(\int_0^{\infty} [x - \alpha w(x)] f_a(x|\alpha) dx + \mu \left[ \int_0^{\infty} \beta \sqrt{w(x)} f_{aa}(x|\alpha) dx - 2 \right] = 0\).

where the term in $\lambda$ cancels out from (5.7) and
\[
\int_0^\infty [x - \alpha w(x)]f_a(x|a)dx = \int_0^\infty x \frac{x - \alpha p}{a^2} f(x|a)dx \\
- \alpha \int_0^\infty \left\{ \frac{\beta^2}{4a^2} \left( \lambda^2 + 2\lambda \mu \left( \frac{f_a}{f} \right) \right) + \left( \frac{f_a}{f} \right)^2 \mu^2 \right\} f_a(x|a)dx \\
= \int_0^\infty x \frac{x - \alpha p}{a^2} f(x|a)dx \\
- 2\alpha \lambda \mu \frac{\beta^2}{4\alpha^2} \int_0^\infty \left( \frac{f_a}{f} \right) f_a(x|a)dx - \frac{\beta^2}{4\alpha^2} \mu^2 \int_0^\infty \left( \frac{f_a}{f} \right)^2 f_a(x|a)dx \\
= \int_0^\infty \left[ \frac{(x - \alpha p)(x - \alpha p)}{a^2} + \frac{\alpha p(x - \alpha p)}{a^2} \right] f(x|a)dx \\
- \frac{\lambda \mu \beta^2}{2\alpha} \int_0^\infty \frac{x - \alpha p}{a^2} \cdot \frac{x - \alpha p}{a^2} f(x|a)dx \\
- \frac{\beta^2 \mu^2}{4\alpha} \int_0^\infty \left( \frac{x - \alpha p}{a^2} \right)^2 \frac{x - \alpha p}{a^2} f(x|a)dx \\
= p - \frac{\lambda \mu \beta^2 p}{2\alpha a^2} - \frac{\beta^2 \mu^2 p}{2\alpha a^3}.
\]

and

\[
\int_0^\infty \beta \sqrt{w(x)}f_{aa}(x|a)dx \\
= \int_0^\infty \beta^2 \frac{(\lambda + \mu \frac{f_a}{f})}{2\alpha} \left( \frac{(x - \alpha p)^2}{a^4} + \frac{ap - 2x}{a^3} \right) f(x|a) \\
= \frac{\beta^2 \lambda a^2 p}{2\alpha a^4} + \frac{\beta^2 \mu}{2\alpha} \int_0^\infty \frac{x - \alpha p}{a^2} \cdot \frac{(x - \alpha p)^2}{a^4} f(x|a)dx \\
+ \int_0^\infty \frac{\beta^2}{2\alpha} \left( \lambda + \mu \frac{f_a}{f} \right) \frac{ap - 2x}{a^3} f(x|a)dx \\
= \frac{\beta^2 \lambda p}{2\alpha a^2} + \frac{\beta^2 \mu p}{\alpha a^3} + \int_0^\infty \frac{\beta^2 \lambda}{2\alpha} \cdot \frac{a p - x}{a^3} f(x|a)dx \\
+ \int_0^\infty \frac{\beta^2 \mu}{2\alpha} \cdot \frac{x - \alpha p}{a^2} \cdot \frac{ap - x}{a^3} f(x|a)dx \\
+ \int_0^\infty \frac{\beta^2 \mu}{2\alpha} \cdot \frac{x - \alpha p}{a^2} \cdot \frac{-x}{a^3} f(x|a)dx \\
= \frac{\beta^2 \lambda p}{2\alpha a^2} + \frac{\beta^2 \mu p}{\alpha a^3} - \frac{\beta^2 \lambda p}{2\alpha a^2} \frac{\beta^2 \mu p}{\alpha a^3}.
\]

Using the above results in equation (5.8) we get

\[
p - \frac{\lambda \mu \beta^2 p}{2\alpha a^2} - \frac{\beta^2 \mu^2 p}{2\alpha a^3} - 2\mu = 0
\]

or

\[
(5.9) \quad p - 2\lambda a - \frac{16\alpha a^3}{\beta^2 p} = 0
\]

where the last equality is obtained by using the expression of \(\mu\).
Multiplying (3.3) by $2a$ and adding it to (5.9) we eliminate $\lambda$ and we obtain a cubic equation for $a$:

$$\frac{p + 4}{p} a^3 + \frac{p^2}{4a} = 0.$$ 

Using (5.3) we may rewrite it as

$$(5.10) \quad (p + 4)a^3 + pUa = \frac{p^2(1 - t\gamma)}{1 + t - t\gamma}$$

and using (3.7) also as

$$a^3 + 3Ka + 2L = 0.$$ 

Since the discriminant

$$D := -108(K^3 + L^2)$$

of this equation is (strictly) negative, our equation has one real and two complex roots. Moreover, since $L < 0$, the real root is (strictly) positive. Finally, this positive root is given by the Tartaglia–Cardano formula (3.5).

**APPENDIX 2: PROOF OF THEOREM 2**

**Proof of (a):** $\frac{\partial a}{\partial \gamma} < 0$. We deduce from (5.10) that

$$\frac{\partial a^3}{\partial \gamma} = \frac{p^2}{p + 4} \cdot \frac{-t^2}{(1 + t - t\gamma)^2} - \frac{p}{p + 4} \frac{\partial a}{\partial \gamma}.$$ 

Since

$$\frac{\partial a^3}{\partial \gamma} = 3a^2 \frac{\partial a}{\partial \gamma},$$

it follows that

$$\frac{\partial a}{\partial \gamma} = \frac{-t^2 p^2}{(1 + t - t\gamma)^2(pU + 3(p + 4)a^2)} < 0. \quad \square$$

**Proof of (b):** $\frac{\partial E(w)}{\partial \gamma} > 0$. Using the equality (5.1) we have

$$E(w) = \int w(x)f(x|a) \, dx$$

$$= \frac{1}{4(1 - t\gamma)} \int \left[ \frac{2a}{p} x + U - a^2 \right]^2 f(x|a) \, dx$$

$$= \frac{1}{4} (1 - t\gamma)^{-1} \left[ \left( \frac{2a}{p} \right)^2 a^2 p + \left( \frac{2a}{p} ap + U - a^2 \right)^2 \right]$$

$$= \frac{1}{4} (1 - t\gamma)^{-1} \left[ \frac{4a^4}{p} + 4a^4 + 4a^2(U - a^2) + (U - a^2)^2 \right],$$

so that

$$(5.11) \quad E(w) = \frac{1}{4p}(1 - t\gamma)^{-1} \left[ (p + 4)a^4 + 2pU a^2 + p(U)^2 \right].$$
Hence
\[
\frac{\partial E(w)}{\partial \gamma} = \frac{t}{4p(1-t\gamma)^2} [(p+4)a^4 + 2pUa^2 + p\langle U \rangle^2]
\]
\[
+ \frac{1}{4p(1-t\gamma)} [4(p+4)a^3 + 4pUa] \frac{\partial a}{\partial \gamma}
\]
\[
= \frac{t}{4p(1-t\gamma)^2} [(p+4)a^4 + 2pUa^2 + p\langle U \rangle^2]
\]
\[
+ \frac{1}{p(1-t\gamma)} \frac{p^2(1-t\gamma)}{1+t-t\gamma} \frac{-t^2p^2}{(1+t-t\gamma)^2[pU + 3(p+4)a^2]} - \frac{p^3t^2}{(1+t-t\gamma)^3[pU + 3(p+4)a^2]}.
\]

It follows that
\[
\frac{\partial E(w)}{\partial \gamma} > \frac{t}{4p(1-t\gamma)^2} [(p+4)a^4 + p\langle U \rangle^2] - \frac{p^3t^2}{(1+t-t\gamma)^3[pU + (p+4)a^2]}
\]
\[
= \frac{apt(1-3t-t\gamma)}{4(1-t\gamma)(1+t-t\gamma)^2}.
\]

We conclude that \(\partial E(w)/\partial \gamma\) has the same sign as \(1 - 3t - t\gamma\).

\[\square\]

Proof of (c): \(\partial E(\pi)/\partial \gamma < 0\). Differentiating the equality

\[(5.12) \quad E(\pi) = \int [x - (1 + t - t\gamma)w(x)] f(x|a) \, dx = ap - (1 + t - t\gamma)E(w)\]

we obtain that
\[
\frac{\partial E(\pi)}{\partial \gamma} = \frac{\partial a}{\partial \gamma} + tE(w) - (1 + t - t\gamma) \frac{\partial E(w)}{\partial \gamma}
\]
\[
= \frac{-p^3t^2}{(1+t-t\gamma)^2[pU + 3(p+4)a^2]}
\]
\[
+ \frac{t}{4p(1-t\gamma)} [(p+4)a^4 + 2pUa^2 + p\langle U \rangle^2]
\]
\[
- \frac{t(1+t-t\gamma)}{4p(1-t\gamma)^2} [(p+4)a^4 + 2pUa^2 + p\langle U \rangle^2]
\]
\[
+ \frac{p^3t^2}{(1+t-t\gamma)^2[pU + 3(p+4)a^2]}
\]
\[
< 0. \quad \square
\]

Proof of (d): \(\partial E(W)/\partial \gamma < 0\). Since
\[
W(x) = x - (1-t\gamma)w(x) + 2(1-t\gamma)^{1/2}w(x)^{1/2} - a^2,
\]
we have
\[ E(W) = \int W(x)f(x|a) \, dx = ap - (1-t\gamma)E(w) + \int \left(\frac{2ax}{p} - a^2 + \overline{U}\right)f(x|a) \, dx. \]

In the last integral the absolute value can be omitted if \( \overline{U} \geq a^2 \). In view of (5.10) this is equivalent to the inequality
\[ p(\overline{U})^{3/2} + (p+4)(\overline{U})^{3/2} \geq \frac{p^2(1-t\gamma)}{1+t-t\gamma} \]
which is assumed in the theorem. Under this assumption we have
\[ E(W) = ap - a^2 - (1-t\gamma)E(w) + \int \left(\frac{2ax}{p} - a^2 + \overline{U}\right)f(x|a) \, dx \]
\[ = ap - a^2 - (1-t\gamma)E(w) + \frac{2a}{p}ap - a^2 + \overline{U}. \]

Using (5.12) it follows that
(5.13) \[ E(W) = E(\pi) + \overline{U} + tE(w). \]

Differentiating with respect to \( \gamma \) and using the expression of \( \partial E(\pi)/\partial \gamma \) and \( \partial E(w)/\partial \gamma \) obtained above, we obtain that
\[ \frac{\partial E(W)}{\partial \gamma} = \frac{\partial E(\pi)}{\partial \gamma} + t \frac{\partial E(w)}{\partial \gamma} \]
\[ = \frac{-t^2}{4p(1-t\gamma)^2} \left[ (p+4)a^4 + 2pUa^2 + p(\overline{U})^2 \right] \]
\[ + \frac{t^2}{4p(1-t\gamma)^2} \left[ (p+4)a^4 + 2pUa^2 + p(\overline{U})^2 \right] \]
\[ - \frac{p^3t^3}{(1+t-t\gamma)^3[pU + 3(p+4)a^2]} \]
\[ \leq - \frac{p^3t^3}{(1+t-t\gamma)^3[pU + 3(p+4)a^2]} \]
\[ < 0. \]

Computation Results for Section 3.1

Table 1: Simulation of effort, \( a \), for \( \theta = 0, \overline{U} = 1.5, p = 3 \)

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
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Table 2: Simulation of expected wage, $E(w)$, for $\theta = 0$, $\bar{U} = 1.5$, $p = 3$

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Table 3: Simulation of expected profit, $E(\pi)$, for $\theta = 0$, $\bar{U} = 1.5$, $p = 3$

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Table 4: Simulation of expected welfare, $E(W)$, for $\theta = 0, \theta = 1.5, p = 3$

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