# PRESENT VALUE AND INTEREST RATES

Many economic decisions involve time in an important way. Students forego current consumption in favor of an education that presumably raises their incomes later on. Firms invest in new plants and equipment that will yield a profit for years into the future. Since so many economic decisions depend on receipts and expenditures that occur in the future, we need to be able to compare dollars today with dollars a year from now, or dollars ten years from now. We develop the necessary tools to make these comparisons in this chapter.

## **Interest Rates**

We begin our study with an example. Let's consider a dollar to be delivered next year. This could be the repayment of a loan to you, or you could be planning on buying something that will cost \$1 next year. What is this \$1 to be paid or received next year worth today? Or to put the question in a slightly different way: how much does \$1 one year from now cost today? When you ask a clerk in a store how much an item costs, you want to know how many dollars you would have to give up **today** to get the item. We apply the same concept of cost to \$1 to be delivered in one year. The cost of \$1 one year from now is the number of dollars we would have to give up **today** to get \$1 next year.

To calculate the cost of \$1 one year from now, you need to know something about interest rates. An interest rate is the rate of growth in the balance of an account or the amount of a debt. For example, suppose that you borrow \$20 from a friend. To payoff the this debt one month later you pay your friend \$21. The \$20 is called the principal and the extra \$1 is called the interest payment. The amount of your debt grew to \$21 over the one month span. The interest rate on this loan is defined as

interest rate = interest payment/principal.

or in this example

1/20 = .05 or 5% per month.

Rates are usually expressed in percentages and at annual rates. For example, an interest rate of 10% means that if you put, say, \$10 into a savings account today and left it there, then one year from now the value of the account would be \$11. In this case the principal is \$10 and \$1 is the interest payment. The \$1 interest payment is 10% of the principal. Though interest rates are typically expressed in percentage form, it is important to remember that when making calculations it is much easier to work with decimals.

To take another example suppose that you borrow \$1000 at an interest rate of 15%. What will be the value of your debt in one year? You will owe the \$1000 principal plus \$150 interest payment. The \$1150 is called the future value of your \$1000 debt or principal. In general, if the principal is \$X and the interest rate is R, then after one year the value will have grown to X + RX. Schematically this may be represented as

 $X = X + R \cdot X = (1 + R)X$ principal interest future value payment

## **Present Values**

With these preliminaries aside, we return to our original question. How many dollars would you have to give up today to get \$1 one year from now? Whatever the eventual answer, for now write it as \$X. Let's assume that the interest rate is 10% so that \$X today will grow into \$(1+.1)X in one year.

We want to know how much X must be today in order for its future value to be \$1. In symbols this question may be expressed as

X(1+.1) = 1

and the solution for X is

 $X = \frac{1}{1+.1} = .9091.$ 

One dollar a year from now costs about \$.91 when the interest rate is 10%. This answer may be put a bit differently. If you put \$.91 into an account earning 10%, in one year the balance in the account would be \$1. The \$.91 is called the present value of \$1 one year from now when the interest rate is 10%. If we think of the \$1 next year as a good similar to a watermelon or a winter coat, the present value is just the price of \$1 next year.

There is nothing special about \$1 or an interest rate of 10%. Instead of \$1 we could use A dollars and instead of 10% we could use R. We can now write the general formula

Present Value of A dollars one year = A/(1+R)from now when the interest rate is R

In words, the present value of A dollars one year from now when the interest rate is R means:

The number of dollars that you would have to give up today to get A dollars one year from now when the rate of interest is R.

Future payments and receipts often occur more than just one year into the future so we need to extend our analysis. It is easiest to begin again with an example. Suppose we are interested in \$1 two years from now, and the interest rate is 10%. The question remains the same: how many dollars would we have to give up today to get \$1 two years from now when we can earn interest at 10%? We know that after one year \$1 will have grown to \$(1+.1), but now we need to know what will happen after two years. To find out, suppose we leave \$1 and the \$.1 interest payment in the account for another year.

At the end of the second year you will receive the principal, which is now (1+.1), and the interest payment on this principal, (1+.1). The future value of 1 two years from now is the 1.1 in principal plus the .11 interest payment or 1.21.

future value two years = (1+.1) + (1+.1)hence of 1

or, if we factor out the (1+.1), we have the simpler expression

(1+.1) + (1+.1) = (1+.1)(1+.1)

When the dollar is allowed to grow for two years it earns interest on the original principal for two years, and earns interest on the first year's interest payment. To see this more clearly multiply out the above expression to get:

$$(1+.1)(1+.1) =$$
 $1 +$  $2(.1) +$  $(.1)(.1)$ 

$$=$$
 \$1 + \$.20 + \$.01

The \$1 represents the original principal, and the \$.20 represents the two interest payments on the original principal. The penny is the interest on the first period interest payment of a dime. It represents the interest on interest. When interest is paid on interest in this manner, it is called compound interest.

We are now ready to find the present value of \$1 to be delivered two years from now. We want to know how much we have to give up today, again this amount X, in order to get \$1 two years from now. In symbols this question may be expressed as

X(1+.1)(1+.1) =

and the solution for X is now

 $X = \frac{1}{(1+.1)(1+.1)} =$ 

To get \$1 two years from now, about \$.83 must be given up today. Thus, the price or present value of \$1 two years hence is about 83 cents when the rate of interest is 10%. Again there is nothing special about \$1, or the interest rate 10% so we can generalize the formula. The general formula for the present value of  $A_2$  dollars to be delivered two years from now is

present value of  $A_2$  dollars two years =  $A_2/(1+R)(1+R)$ from now when the interest rate is R

This may be written in a more compact form:

present value of  $A_2$  dollars two years =  $A_2/(1+R)^2$  from now when the interest rate is R

In words the present value of  $A_2$  dollars two years from now when the interest rate is R means:

The number of dollars that you would have to give up today to get  $A_2$  dollars one year from now when the rate of interest is R.

We could continue on and consider the case of \$1 to be delivered three years hence, but this is unnecessary now. The previous formula establishes a pattern, and we are spared the tedium of having to develop the formula for three years from scratch. Instead we can make the educated guess that the general formula for the present value of  $A_3$  dollars three years from now is:

present value of  $A_3$  dollars three years =  $A_3/(1+R)^3$  from now when the rate of interest is R.

and in words this means:

the number of dollars that you would have to give up today to get  $A_3$  dollars three years from now when the interest rate is R.

We can now quickly state the formula for an arbitrary number of years. We can write:

present value of  $A_n$  dollars n years from  $= A_n/(1+R)^n$  now when the rate of interest is R.

You are welcome to work out the meaning in words for yourself.

## The Present Value of a Stream of Payments

Often times we will consider not just one future payment or receipt, but instead a stream of payments or receipts that may occur over many years. For example, consumption and income occur over many years, not just one or two years from now; and the same can be said for profits from investment projects or income from savings. The present value of a stream of payments is the sum of the present value of each payment. Suppose the stream of payments is \$1 next year and another \$1 two years from now. The present value of this stream of payments is just:

Present value of the stream of payments \$1 one year from now and =  $1/(1+.1) + 1/(1+.1)^2$ \$1 two years from now when the rate of interest is 10%

and this equals

$$.91 + .83 = 1.74$$

In words this means that to get \$1 next year and \$1 two years from now, you would have to give up \$1.74 today.

There is still nothing sacred about the \$1 figure or the interest rate of 10%, so we can go ahead and write out the formula for the present value of the stream of payments  $A_1$  and  $A_2$  when the interest

rate (in both periods) is R. It is

present value of the stream of dollar payments  $A_1 = A_1/(1+R) + A_2/(1+R)^2$ and  $A_2$  when the rate of interest is R

Of course the real world will give us cases where the stream extends out over many years. House payments often extend over a period of 25 or 30 years, and the economic life of a factory may be equally long. Fortunately a pattern has presented itself, and we do not have to trudge through the details for the 3-year case, the 4-year case, and so on. We can guess from the formula for the 2-year case that the general formula is

 $PVSTREAM = A_1/(1+R) + A_2/(1+R)^2 + ... + A_n/(1+R)^n$ 

where PVSTREAM is short hand for the present value of the stream of payments  $A_1$  dollars one year from now,  $A_2$  dollars two years from now, and so on until  $A_n$  dollars n years from now. In words it means the number of dollars that you would have to give up today to get the stream of payments  $A_1$ ,  $A_2$ , ...,  $A_n$ .

## Applications

The present value formula derived above has three parts. These parts are:

- 1) PVSTREAM
- 2) the interest rate R
- 3) the stream  $A_1, A_2, \dots A_n$

Broadly speaking, given any two of these parts the third one can be calculated from the formula. The particular situation or application determines which of the three are given, and which one remains to be found. We give brief examples of the three possibilities.

#### a. car payments

Suppose you are shopping for a new car. The dealership you visit is offering low interest rates, say 2%. This gives you R. The price of the car, which is just the number of dollars you have to give up today to get the car, is quoted on the sticker. This is the PVSTREAM. If the car loan is a 5-year loan and all the payments are of equal size, then all that remains is the calculation of the five equal payments  $A_1, A_2, ..., A_5$ . In this case this stream would be your annual car payments.

#### b. lottery payments

Now imagine that you are the treasurer for Ohio and someone has just won the lottery. The state is obliged to pay the winner, say, \$2 million a year for the next twenty years. This is the stream of payments. Suppose the 20-year interest rate is 8%. How much of the revenue from lottery ticket sales must the state put away to pay the lottery winner? In this case you have the interest rate and the stream of payments. The PVSTREAM remains to be calculated.

#### c. coupon bonds

A very important application for our purposes requires the calculation of the interest rate. A lot of the borrowing and lending that takes place in an economy takes place in the form of buying and selling so-called coupon bonds. A picture of a coupon bond is drawn in Figure 5.1.

The bond has several critical features. First, the name of the issuer is on the bond. The bond is a promise by the issuer to pay the face value of the bond on its maturity date. In our example (the parts in brackets) the bond is a promise by the U.S. Treasury to pay the holder of the bond \$100 on January 1, 2002. This is not the bond's only promise. In addition to the face value payments, the issuer promises

Figure 5.1				
A Coupon Bond				
Name of Issuer (U.S. Treasury)				
Face Value (\$100)Coupon Rate (7%)Maturity Date (1/1/02)				
Coupon Payment (\$7)	Coupon Payment (\$7)			
Maturity Date Coupon Payment	Maturity Date Coupon Payment			
(\$7)	(\$7)			
Maturity Date	Maturity Date			
Coupon Payment (\$7)	Coupon Payment (\$7)			
Maturity Date	Maturity Date			

to pay the holder a series of coupon payments. In our example the payments are \$7 on the New Years days of 1997 through 2002. The amount of the coupon payment is determined by the coupon rate and the face value, and equals in our example

 $7 = (.07) \cdot (100),$ 

or more generally

coupon payment = (coupon rate in decimal form)(face value).

The coupon rate is stated on the bond, and once the bond is issued it cannot change.

Coupon bonds are bought and sold in organized markets. For example, there is a large and active market in government bonds. If you cared to buy a coupon bond, you could call a treasury bond dealer, and, given the coupon payments and face value of the bond, you could make an offer. Similarly, if you currently owned a U.S. treasury bond and wanted to sell it for cash, you could call a dealer and ask for a bid. In short, treasury bonds are widely traded every day, and in the process of trading a price for these bonds is established; we call it P<sup>b</sup> for short.

The bond offers the stream of payments CP,...,CP, F to its holder where CP is a coupon payment and F is the face value payment. The price of the bond is nothing more than the number of dollars you would have to give up today to get the stream of payments CP,...,CP, F; and is just the PVSTREAM of the coupon and face value payments. The interest rate R is left to be calculated. According to the formula, the interest rate R, which is also called the **yield to maturity** of the bond, is the rate that makes the present value of the coupon and face value payments equal the price of the bond. In symbols the yield to maturity is the rate R that makes the following equation true:

$$P^{b} = \frac{CP}{(1+R)} + \frac{CP}{(1+R)^{2}} + \frac{CP}{(1+R)^{3}} + \bullet \bullet \bullet + \frac{CP+F}{(1+R)^{n}},$$

where n is the number of years, or term, to maturity, 5 in our example.

Several of the properties of this interest rate or yield are of interest to us. Suppose the price of the bond falls. What happens to the rate R? Well, since the coupon and face value payments are fixed, when P<sup>b</sup> falls the rate R must rise for the above equation to remain true. This means that when the price of the bond falls, the interest rate on it rises. Similarly, when the price of a bond rises, its interest rate or yield falls. There is thus an inverse relationship between bond prices and interest rates. This relationship is a matter of definition and is not dependent on any particular theory being true.

We use the inverse relationship between bond prices and interest rates throughout the analysis. It is therefore very important to become comfortable with it. We have already been through the formal definition, and we should spend a moment on its simple intuition. A bond is a promise to pay a fixed stream of dollars. When the price of the bond falls it means that you have to give up fewer dollars today to get the same stream of payments. This is clearly a good deal for the purchaser, and means that your return, or interest rate, is higher. Lower bond prices raise the return from holding the bond. If you have to pay more money for the same stream of payments, this makes you worse off and lowers your yield. Higher bond prices lower the return from holding the bond.

It is sometimes easy to confuse the coupon rate and the interest rate, but in general the two are not the same. There is one exception when the two will coincide. This occurs when the price of the bond equals its face value. When this happens the bond is said to sell at par. However, when the price of the bond is below its face value, the bond is said to sell at a discount or below par, and the interest rate exceeds the coupon rate. To see why the yield or interest rate exceeds the coupon rate, suppose the bond in our example sells for a discount at \$95. The holder receives a \$7 payment each year which is a return of \$7/\$95 = .074 or 7.4%. If at the time of maturity the holder received the \$95 that he paid for the bond, then he would have earned a 7.4% yield over the period. But instead of \$95, the holder will receive more. This \$5 "extra" is called a capital gain and lifts the yield or return on this bond to about 7.5%, one half of a percentage point above the coupon rate. In this case the bond sells at a premium or above par, and the interest rate or yield will be below the coupon rate.

#### d. stock prices

Another application that is very important deals with how stocks are priced. A share of common stock represents part ownership of a corporation. How large a part of the corporation is owned depends on the total number of shares outstanding relative to the number of shares the stockholder owns. For example, suppose there are one million shares outstanding and the stockholder in question owns ten thousand shares. This stockholder owns 10,000/1,000,000 = .01 or 1% of the corporation. With ownership come some privileges. Two important privileges are a vote at the annual meeting, where the board of directors of the corporation is elected, and a right to share in the profits of the corporation. Both rights depend on the portion of stock held. So, our stockholder would have 1% of the votes at the annual meeting and a claim to 1% of the corporation's profit.

If an individual held all the stock of a corporation, she would have sole ownership. As sole owner she would have claim to all the corporation's current and future profits. We can write the present value of this profit stream as

PV of corporate profit stream = corprof<sub>t</sub> + corprof<sub>t+1</sub> /(1+R) + corprof<sub>t+2</sub> /(1+R)<sup>2</sup> + ...,

where  $corprof_t$  is corporate profits at time t and these profits may be earned into the indefinite future. This present value is called the fundamental value of the firm. How much did they have to pay to lay claim to this stream, this fundamental value? If there are one million shares of stock and the price of each share is \$15, then the cost of the firm is \$15 million. The cost of the firm is just the number of dollars that you would have to give up today to get the stream of corporate profits; in other words, the present value of the stream of corporate profits. So, we have

$$P^{\text{stock}} \bullet \# \text{ of shares} = \text{corprof}_t + \text{corprof}_{t+1} / (1+R) + \text{corprof}_{t+2} / (1+R)^2 + \dots$$

This formula reveals the basic determinants of stock prices. Other things the same, if the corporation announces a new product that market analysts believe will be a highly profitable line, forecasts of corporate profits will increase; and the stock price will rise. On the other hand, if news leaks that the Justice Department is about to bring a lawsuit against the corporation, profit forecasts will fall, and so will the price of stock. If interest rates rise, the fundamental value of the stock falls, and so does its price. Finally, if the corporation issues more stock, for example if it "splits" its stock by giving each investor additional shares, then the price of the stock will fall. In the case of a stock split of two new shares for one existing share, the price will be cut in half.

It is important to keep in mind that future corporate profits are very difficult to forecast. This means that risk plays an important role. So, if the corporation is a new venture with an uncertain future, the interest rate with which you discount ought to be large to take this into account. Also, difficulty in forecasting leads to disagreements, and this may cause the price of a stock to wander from its

fundamental value, if the crowd becomes overly optimistic or pessimistic. In short, the above pricing formula is a beginning to the analysis of stock prices, not the end.<sup>14</sup>

### **Inflation and Interest Rates**

Inflation, or more precisely the expectation of inflation, has important implications for interest rates. For example, suppose households expect prices to stay the same, then a household that lent \$1 expects to be repaid with dollars that have the same purchasing power. When inflation is expected this changes. Suppose a bundle of goods today costs \$10, but you expect the same bundle to cost \$11 next year. The expected rate of inflation is 10%. This means that the value, that is to say the purchasing power, of the dollar is declining. Today \$1 will buy 1/10 of a bundle, but next year the same \$1 will buy only 1/11 of a bundle. The purchasing power of the dollar has fallen by 10%. How does the loss of purchasing power of the dollar affect interest rates? To answer this question consider the following example.

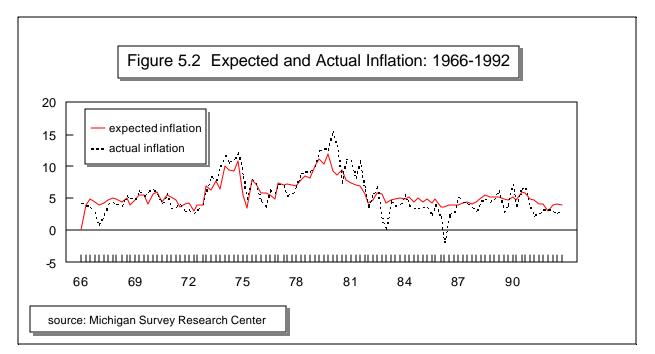
Suppose you are willing to lend someone 10 bundles of goods today in exchange for 11 bundles of goods next year. The interest rate in terms of bundles in 10%, and, since it is in terms of bundles, this rate is called the **real rate of interest**. Now suppose the cost of a bundle of goods today costs \$100, and you expect the cost of the bundle to still be \$100 next year. This means you don't expect any inflation. The loan of 10 bundles today for 11 bundles next year is equivalent to a loan of \$1000 (= $$100 \cdot 11$ ) next year. The interest rate in terms of dollars is 10%, and, since it is in terms of dollars, it is called the **nominal rate of interest**. When there is no expected inflation, the real rate of interest and the nominal rate of interest are the same.

Now we introduce expected inflation. Suppose the cost of a bundle of goods today is still \$100, but next year you expect the bundle to cost \$120. You expect inflation of 20%. Now reconsider the trade of 10 bundles today for 11 bundles next year. The real rate of interest remains at 10%, but the nominal rate must change. In dollar terms the loan now becomes \$1000 (= $$100 \cdot 10$ ) today for \$1320 (= \$120 \cdot 11) next year. The dollar interest payment must increase by \$220 to keep up with inflation.

<sup>&</sup>lt;sup>14</sup> For a good discussion of stocks and their prices written for the lay person see Burton Malkiel's *A Random Walk Down Wall Street* (1985).

This makes the nominal rate of interest 32%. Expected inflation has driven a wedge of 22 percentage points between the real and nominal rates of interest.

What accounts for these changes? When inflation is 20%, the purchasing power of the dollar falls at this same rate of 20%, and this means that the value of the \$1000 principal falls by 20% or \$200. So, \$200 of the \$220 increase and 20% of the new nominal rate of interest reflect compensation for the loss of purchasing power of the principal. But the principal is not the only payment that is losing purchasing power. In the absence of expected inflation the interest payment was \$100, which reflected the 10% real rate of interest, but the purchasing power of the \$100 interest payment is also falling. To regain the lost purchasing power of the interest payment, the lender requires 20% of \$100, or \$20. This \$20 accounts for the remainder of the higher interest payment. The \$20 increase is 20% of the 10%



interest payment and also accounts for remaining 2% (= 10%•20%) increase in the nominal interest

### rate. To summarize

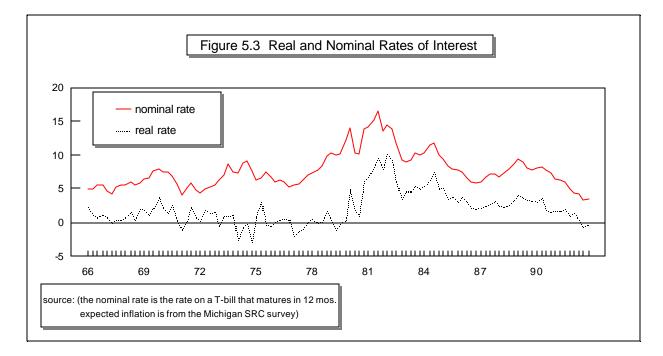
interest rate:	32% =	10% +	20% +	2%
	nominal return	real return	compensation for lost purchasing power of the principal	compensation for lost pur - chasing power of the inter- est payment
interest payment:	\$320 =	\$100 +	\$200 +	\$20

In general we let  $R_t$  be the nominal rate of interest at time t,  $r_t$  be the real rate of interest at time t, and  $\pi_{t+1}^e$  be the rate of inflation expected to prevail over the period t to t+1. The general relationship between the nominal and real rate of interest is given by

$$R_{t} = r_{t} + \pi_{t+1}^{e} + r_{t}\pi_{t+1}^{e}$$

In many cases the multiplicative term (10%•20% in our numerical example) is dropped, and the following approximation is used

$$R_t = r_t + \pi_{t+1}^e.$$



The original equation and its approximation are often called **Fisher's equation** after Irving Fisher, an important economist in the first half of this century who studied this equation and made it famous.

For lenders the real return determines the increase in their standard of living gained by saving. For borrowers the real return determines the cost of borrowing today in terms of future goods forgone. It is thus the real rate of interest that ultimately influences behavior. If  $\pi_{t+1}^e$  is zero, then the real and nominal rates of interest are the same; and the distinction between the real and nominal rates is unimportant. However, in an environment of inflation it is essential to understand the difference between the two. The nominal and real interest rates need not move in the same direction. In late 1979 nominal interest rates, measured by the short term treasury bill rate, were about 11%, but at the same time expected inflation, measured by surveys taken at the time, were about 10%. This means the real rate on short term treasury bills at the end of the 1970s was about 1%. By the middle of 1987 nominal interest rates had fallen to about 6.5% so some 4.5 percentage points had been trimmed from the nominal rate. However, expected inflation had fallen by even more to around 4%, a decline of 6 percentage points, and as a result the real rate of interest in the middle of 1987 had risen to about 2.5%. In this case, lower nominal rates did not translate into lower real rates.

To calculate the real rate of interest you must have a measure of the expected rate of inflation. The Michigan Survey Research Center provides us with such a measure. Each quarter they ask households what they expect inflation will be over the coming year. This expected rate of inflation along with the actual rate are plotted in Figure 5.2. You can see from this picture that the public makes errors. They tended to underestimate inflation in the 1970s and overestimate inflation in the 1980s, particularly the early '80s. Nevertheless, their predictions are reasonable and on average close to the mark.

The real rate of interest based on the Michigan-SRC expected rate of inflation is shown in Figure 5.3. The nominal rate of interest is also plotted there. In general, it appears the real and nominal rates move together, but there are exceptions. Note, for example, in 1974 when real rates rose at the same time nominal rates fell. The graph also reveals that real interest rates were high in the 1980s when compared to their levels in the 1960s and 70s.

## Summary

In this chapter we introduced the idea of present value. This concept allows us to compare payments that occur at different times. There are many applications of present value, and we just touched on a few. For our purposes it is important to remember the inverse relationship between interest rates and bond prices that is implied by present value relationship, and the distinction between real and nominal rates of interest.

## **REVIEW QUESTIONS**

- 1. a) Suppose that the face value of a bond that matures in one year is \$100 with a coupon rate of 8%. You can buy this bond today for \$95. What is the interest rate (yield to maturity) on this bond?
  - b) What happens to the interest rate on the bond, if its price falls to \$92?
- 2) Suppose you buy a car for \$5000. You borrow the entire amount on a 3-year fixed rate loan. That is, a loan that is completely repaid in 3 years and the interest rate on the loan is fixed at 8% per year. The payment schedule calls for three equal payments beginning in one year. How much is each payment?
- 3) To get an approximation of how much monthly payments would be with this loan divide your answer to 2 by 12.
- 4) The winner of the Ohio lottery receives 20 checks in equal amounts. The first check is paid at the time the winner is announced. For example, if the jackpot is \$20,000,000 the winner receives 20 checks, one per year, for \$1 million. The first payment is made shortly after the presentation of the winning ticket. If the lottery jackpot is \$10 million, and the current interest rate is .07, what is the present value of the jackpot? Is it really a \$10 million jackpot?
- 5) What is the present value of \$312 to be paid 10 years from now when the interest rate is 12%? Give both a numerical answer and the intuitive or conceptual meaning.
- 6) Consider the following two contracts. To keep things simple suppose the dollar amounts are paid once year.

contract A	year1	year2	year3
	\$300,000	\$400,000	\$500,000
contract B	\$200,000	\$600,000	\$400,00

If you are an agent for a pro athlete, and the interest rate was 10%, would you recommend contract A or contract B? Explain.