Annualization

Suppose at date \( t_1 \) you lend someone $100 and at date \( t_2 \) the borrower pays you $101. You earn 1\% interest. Is this interest rate high or low? If \( t_1 \) is 2000 and \( t_2 \) is 2010, then you have lent your money very cheaply. On the other hand, if \( t_1 \) is January 1 and \( t_2 \) is January 2 of the same year, then you have done very well indeed. In short, whether an interest rate is high or low depends critically on the time interval. To avoid the ambiguity associated with an ignorance of the time interval, interest rates are quoted on an annualized basis.

What does annualized basis mean? Suppose in our example above \( t_1 \) was April 24, 2004 and \( t_2 \) was six months later, October 24, 2004. To annualize this rate, we ask to what value our dollar would grow if it kept growing for another six months at the same rate. At the end of the first six month period the balance has grown to $1.01. If over the next six months, which will make a year, the balance grows at the same rate you will have \((1 + .01)\times1.01 = 1.0201\). The annualized rate of interest is 2.01\%. It is important to see that the annualized rate is not just two times the six month rate. The .01\% points represents compounding, and compounding can be very important over many periods, or when the rates are high, or when amounts borrowed are large.

To summarize, let the interest rate over 1/2 a year be written as \( R_{1/2} \). The annualized rate of interest is just

\[
1 + R_A = (1 + R_{1/2})(1 + R_{1/2}) = (1 + R_{1/2})^2.
\]

If the six month rate had been, say, 6\%, then the annualize interest rate would have been found by calculating

\[
1 + R_A = (1 + .06)^2 = 1.1236.
\]

The annualize rate is just 12.36\%. Note that on a loan of $10,000,000 compounding represents $36,000 in interest payments; not a trivial sum.
The procedure that we just derived for loans of half of a year works in a similar way for loans for other fractions of a year. For instance for loans of three months, a quarter of a year, you annualize in the following way:

\[ 1 + R_A = (1 + R_{1/4})(1 + R_{1/4})(1 + R_{1/4})(1 + R_{1/4}) = (1 + R_{1/4})^4. \]

So, if the three month interest rate is .02, the annualized rate of interest is 8.24% since \((1.02)^4 = 1.0824\) after some rounding. In general, if an interest rate is for a loan over \(1/N^{th}\) of a year, then you calculate the annualized rate using

\[ 1 + R_A = (1 + R_{1/N})^N. \]

This formula can also be used to annualize interest rates on loans that last of more than one year. For example, suppose you lend $100 to a borrower on April 24, 2004 and the lender repays $110 on April 24, 2006. In this case \(N = 1/2\) and \(R_{1/N} = R_2 = .1\), and you find the annualized rate using

\[ 1 + R_A = (1 + .1)^{1/2} = 1.0488, \]

so the annualized rate is 4.88%.

The federal funds rate is the interest rate that one bank charges another on an overnight loan. Even though the loan is literally overnight, the federal funds rate is quoted on an annualized basis. Suppose the annualized federal funds rate is 5% and a bank borrows $10,000,000 from another. What will the interest payment be on this overnight or one day loan? To answer this question we need to know the daily interest rate. Since a day is \(1/365^{th}\) of a year, we have the relationship

\[ 1 + .05 = (1 + R_{1/365})^{365}. \]

From your high school math, you know that
(1 + .05)^{1/365} = 1 + \frac{R}{365}

or

1.000134 = 1 + \frac{R}{365},

so

.000134 = \frac{R}{365}.

The interest payment can now be calculated. It is

.000134 \times 10,000,000 = $1340.

**Conclusion**

In order to easily allow comparisons, interest rates are quoted on an annualized basis. However, if you want to calculate the dollar payment you will receive in, say, three months on a loan of $500,000 at an annualized interest rate of 7%, you must be able to "unpack" the annualization as we have done above with the federal funds loan.