

*Maria Eugenia Rodrigues Araujo-09/03/99- register n. 176.053.e-mail:
mareuge@zipmail.com.br Universidade Federal do Maranhão-Brazil.*

ABAS - ACADEMY OF BUSINESS & ADMINISTRATIVE SCIENCES

1999 International Conference-Spain July 12-14

Program Theme:

Globalization & Emerging Economies

Cost-Volume-Profit Analysis Using The Monte Carlo Method.

Nowadays the world is readying itself for Globalisation, it is fundamental that countries with emerging Economies are internally prepared to participate with countries that are in the same trading zone.

Countries with emerging economies suffer at times from the effects of political and economic events wich leave their businesses in a state of uncertainty. Within these variables wich are reflected in the price of the product or service and consequently in the result.

To best administrate businesses in this scenario they must be equipped with more efficient administrative controls wich minimise the risk for investment.

With this in mind this study is concerned with the use of the technology of the Quantitative Method, specifically in the case of the Generation of Random Numbers which can simulate different Break-even-Points in the activity of the business along with C-V-P Analysis, therefore embellishing the accounts information available to the administrators of the Business.

In this way the Monte Carlo Method is used as a tool that enables the simulation of uncertainty possible, uncertainty which can occur within the sales processes of the business, using the generation of random numbers for variables of the Cost-Volume-Profit relation to better the Analysis of the Breakeven-Point.

The study of the article Analysis of Cost-Volume-Profit under uncertain conditions (Jaedicke;Robichek-1964) serves as a base for this research. they worried about verifying uncertainty using the concept of normal distribution applied in a distribution of sales.

2 – Objective

The objective of this research is to apply the Monte Carlo Method in the analysis of Cost-Volume-Profit, verifying possible locations that the Breakeven-Point can have and therefore allowing a better evaluation of the intrinsic Risks for a Project or Business.

3 – Problem

To investigate the application of the Monte Carlo Method within the behaviour of Fixed Cost Variables, Price and Variable Cost to obtain the new Breakeven-Point.

4 – Hypothesis

To understand the objectives of this research the following Hypotheses were taken as standard.

- a) The Distribution of variables are already known.**
- b) The Monte Carlo Method allows the simulation of values that the Breakeven point can have.**

The verification of these hypotheses requires research that allies Technical Statistics to Accountancy, along with accountants and administrators having information that enables them to make decisions in the long and short term.

5 – Applied Methodology

The following aspects are considered in the development of this Work.

1 – Initially the study of the concept of analysis cost-volume-profit. (C-V-P).

2 – Procedures in the Analysis Cost-Volume-Profit under conditions of uncertainty. (Jaedicke,Robichek –1964).

3 – The functioning and application of the Monte Carlo Statistical Method Using Soft Excel 7.0 In the processing of Data. This software makes the process easier.

4 – Results and statistical interpretation.

5 – Conclusions.

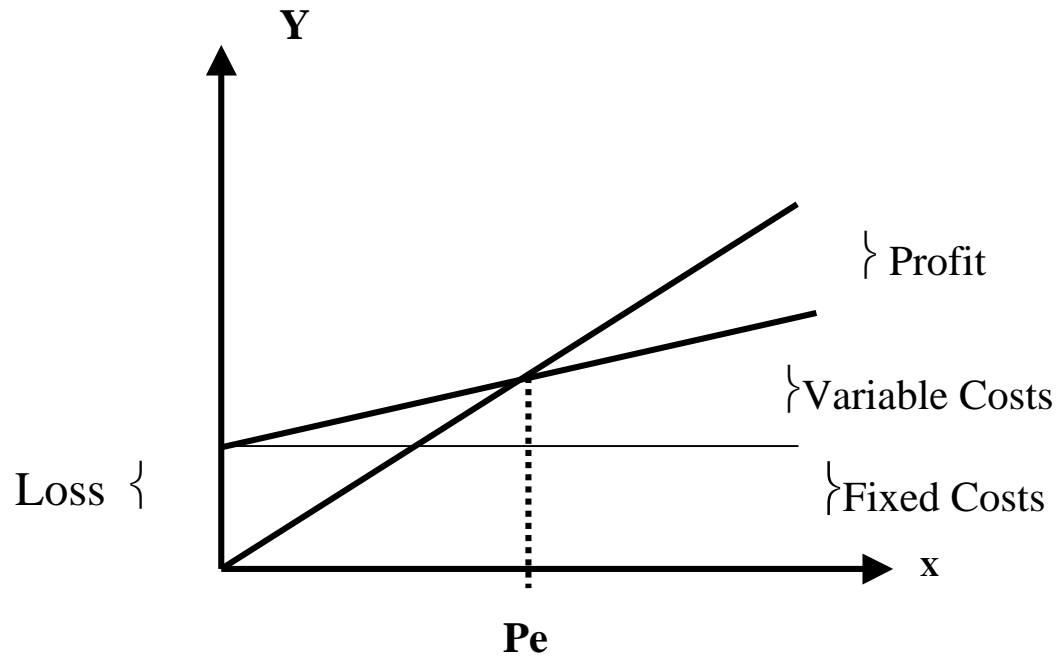
1 – COST-VOLUME-PROFIT ANALYSIS

The Analysis of Cost-Volume-Profit is an instrument used by the Administration of a Business in the area of production. It allows the visualisation of incurred costs in the process of production of goods or services. How the quantity produced effects profit and

or harms the generation in the interaction of these variables at the time they were verified. The Cost-Volume-Profit Analysis favours a reduction in Fixed Costs/and or Variables, increasing the volume of production and defining price and volume of sales as a projection of Profit.

Consider the Cost-Volume-Profit relation as the Linear Mathematical Function $P_x = V_x + C_f$ and plot the data extracted from this function, where the Axis X in the abscissas represents the quantity of production and the Axis Y represents the monetary value. This method facilitates the interpretation of Variable Costs, Total Costs and the receipts from the product on sale.

On this graph note the point of intersection of the lines, this is the Equality of Total Cost and Income, where profit is zero and serves as an indication to where the administrator can make a reading and then determine the necessary volume for Equilibrium. The value above this volume is profit for the business and the value below is the value where the equilibrium shows that the business is operating at loss. In fact earnings or losses originate from uncertainty of these variables as shown in the graph.



Graph 1 – Break-Even Point

2 – Cost-Volume-Profit Analysis Under Conditions Of Uncertainty.

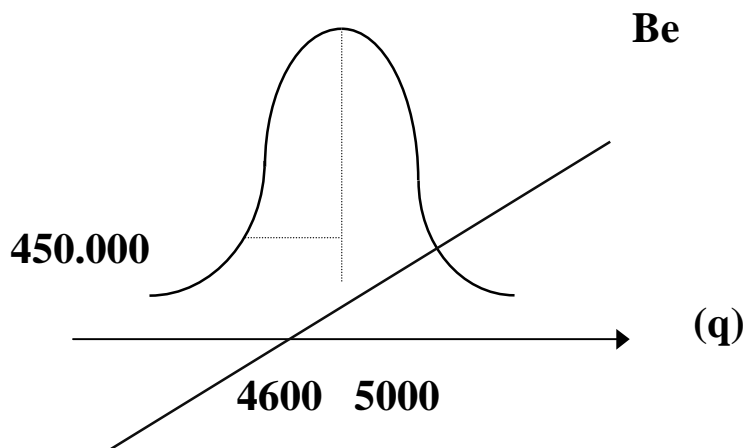
The traditional opinion of Jaedicke and Robicheke “Analysis Of Cost-Volume-Profit Under Uncertain conditions” Focuses on the variables of the C-V-P analysis under uncertain conditions understood to be an approximation. for this Jaedicke And Robitichek used the concept of random variables in the data. For the value of this random variable helps in the decision, when applied, to estimate the degree of uncertainty of the variable in

the study. This is, to know the probability of assumed possible values within a Distribution.

The objective is to choose which random variable gives the best profit expectation.

Graph and figures follow.

figure 1 break-even point



be = breakeven point

q = quantity of sales.

So, the expected profit can be given by:

$$E(Z) = E(Q) (P-V) - F$$

$E(Z)$ = Expectation of Profit

E(Q)= Expectation of Sales

P = Price

V = Variable Cost

F = Fixed Costs

the standard deviation of profit is:

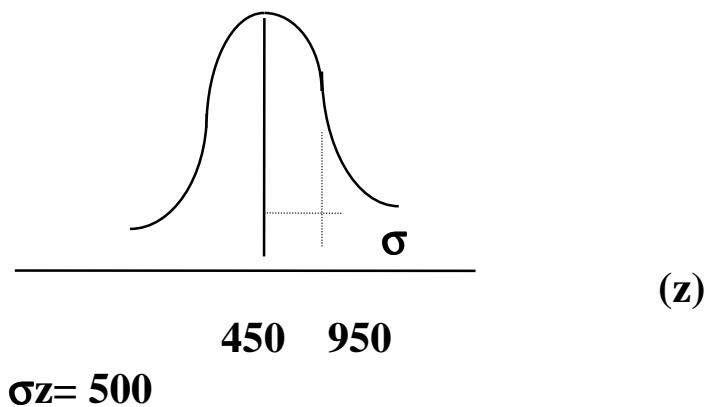
$$s_z = s_q \times mC =$$

utilising the data of Jaedicke and Robichek we have:

$$cm = 3000 - 1750.$$

$$\text{hence: } s_z = 400 \times 1,250 = \$ 500,00.$$

figure: 2 standard deviation of profit



This example shows the relationship between the level of profit and the distribution of the probability of the level of profit. A number of important relationships can now be obtained in terms of probable. Supposing the probability of distribution of the quantity sales is normal with a mean of 5000 units and standard deviation 400 units, the probability of the distribution of profit can also be normal with an average of \$450,000 and a standard deviation of \$500,000.

The example developed by Jaedicke and Robicheke, demonstrates that it is possible for the administrator to know, at least, the probability of reaching the breakeven-point. when the possibility exists of the profit being greater than zero, zero can be calculated assuming the area below the distribution to the right of the profit is equal to zero. this probability is calculated considering $1 - (\text{the probability of profit is less than zero})$ when the distribution is normal and adopts the table z, to determine

the area beneath the curve along with the given value of z.

for example, consider the given level of sales is \$450 and the standard deviation is \$500 consequently :

$$\frac{450 - 0}{500} = 0.9$$

which corresponds to 0.184, so the probability of profit is less than zero and

$$P(\text{Profit} < 0.9s) = 0.184$$

$$1) \text{If } P(\text{Profit} > 0) = 1 - 0.184 = 0.186$$

$$2) \text{If } P(\text{Profit} > \$200,000) = 1 - P(\text{Profit} < \frac{450 - 200}{500}s)$$

$$1 - P(\text{Profit} < 0.5\sigma) = 1 - 0.3085 = 0.692$$

$$3) P(\text{Loss} > \$300,000) = P(\text{Loss} > \frac{450 - (-300)}{500} \text{ or } 1.5s)$$

$$P = 0.067$$

This thought leads the business to assume the risk of loss or increase the profit since this decision is calculated for this type of analysis adopting statistical concepts to know probable risks and uncertainties. From the example of Jaedicke and Robicheke the following is extracted

fixed costs = \$5800

sale price = \$3000

variable cost per unit = \$1750

The break-even point will be

be = $\frac{\$5800}{\$3000 - \$1750}$ = 4640 units

\$3000 - \$1750

From Jaedicke and Robicheke in the analysis of cost-volume - profit can be imagined at the normal distribution associated on the graph of c-v-p analysis.

We can now apply the Monte Carlo simulation model.

3 - Functioning and Application of the Statistical Monte Carlo method using soft excel to process the data.

Diverse authors like Freitas, Corrar and Saliby, Hertz studied the application of the Monte Carlo Method.

Freitas in evaluation the fundamental analysis using the monte carlo method comments that this is a tool that can be used to expand complex projects and make provisions. he presents the advantages of the method as follows.

- a) the majority of complex systems with statistical elements cannot be described correctly by deterministic models. with this type of problem the monte carlo method could be a solution.**
- b) it permits the estimation the performance of a system under diverse projected conditions.**
- c) increasing the political alternatives for a unique system of systems, alternatives can be compared with simulation.**

Freitas enlarged more that selecting distributions of probability of variables entered and determining this data within a given model..

For Freitas, random uniform numbers compose the backbone of the generation of random uniform numbers. the monte carlo (MC) method has as a base a random sample following principles similar to roulette to generate results and distributions of probabilities corresponding to this result..

Agrees (Sobol'83) cited research by Corrar, the objective of the MC method is to simulate the behavior of processes which depend on random factors. the monte carlo method allows a better view of the specimen, once clearly confirmed that \bar{x} is a reliable estimate of μ , in a complex mathematical situation. this is the most practical way to determine the sample distribution says Wonnacott.

There are 4 stages of execution of the monte carlo method on computer according to Wonnacott.

- 1) Monte carlo functions like roulette on the computer. so its chosen in a table with z as a standardised value.**

2) make the conversion of z by the formula $z = \frac{\bar{x} - \mu}{\sigma}$

3) Repeat the process in conjunction with desired observations and therefore the average \bar{x} is computed.

4) In the same way the averages of the sample \bar{x} are computed and arranged on a graph of the distribution of the sample \bar{x} .

The next step is to test the montecarlo method on the breakeven point of distribution of sales to verify the effect presented at different results on the breakeven-point.

Excel generates random numbers which were tested in the models of Kolgomorov-Smirnov, chi-square is an autocorrelation to verify the generation of numbers. the results did not find evidence that the generation of random numbers isn't the normal didtribution in all of the variables entered.

4 Methodological Procedures

The monte carlo method can be applied to the c-v-p model whose collected datas are extracted from Jaedicke and Robicheke cost-volume-profit analysis under uncertain conditions of uncertainty in which an analysis of the distribution of sales is considered a normal distribution and which is verified by research.

5 Collated Data

In one distribution of sales with an average of 5000 and a standard deviation of 400, the breakeven-point is given by the formula

$$PE = \frac{CF}{P - C_v}$$

resulting in 4.640, displayed below

$$PE = \frac{5800}{3000 - 1750} = 4640$$

fc =fixed cost=5800,

p=price=3000

vc= variable cost=1750

5 Processed Data

With excel the calculated breakeven-point is made through the creation of the formula Be, and the data by simulating the MC are calculated by the function of generation of random numbers. with this information 500 numbers are generated with each variable cfc,p,vc and from each sample one average is extracted. this is repeated 100 times.

this process gave

for FC an average of 528,977

for P an average of 2965,84

for cCV an average of 2094,417

These values were substituted in the formula on the breakevenpoint and a new break-even point was found. therefore the stable break-even point was found at 6.689036.

To compare this new break-even point starting at 4.460 was it necessary to know the necessary volume of sales?

To answer this the rule of three simples was applied and a volume of 7.208 unit was found. this result can be seen on the graph of distribution of sales presented by Jaedicke and Robitchek (fig.1).

From the sample of simulation of 100 diferente of breakeven-point an average break-even point was simulate (bems) equal to 6,890591 and taken from this same sample, a standard deviation of 1,487457 which can be compared to the initial break-even point given at 4,640 at which the interval is inserted.

between this was adopted the premise of

$$6,890591 \pm 2\sigma$$

to obtain with more certainty the interval 2σ , which corresponds approximately to 95% certainty that it is equal to 2,974914, interval of $6,890591 \pm 2,94914 = (3,915677$ and $9,865505)$.

the same procedure was adopted for the generation of 1000 numbers and the results obtained were:

for 1000 cf = 6032,92841

for 1000 p = 3051,79345

for 1000 cv = 1574,45843

The stable breakeven-point was met at 4,083365626

in the same way the rule of three was applied to know which volume of sales for this stable breakeven –point gives the value of 4400 .

100 breakeven-point were simulated and an average extracted equal to 4,672955 and the standard deviation that is equal to 0,622027756. the standard deviation is used together with the simulated pe to know the interval at which the initial Be is situated in relation to the simulated Be. For the premise of $4,672955 \pm 2\sigma$ that resulted in : $4,672955 \pm 1,24405512 = (3,42886543 \text{ and } 5,91697567)$.

5 –Results

The results obtained in the Monte Carlo method confirmed that:

1 – the random generation of variables involved in cost –volume-profit analysis present a normal distribution.

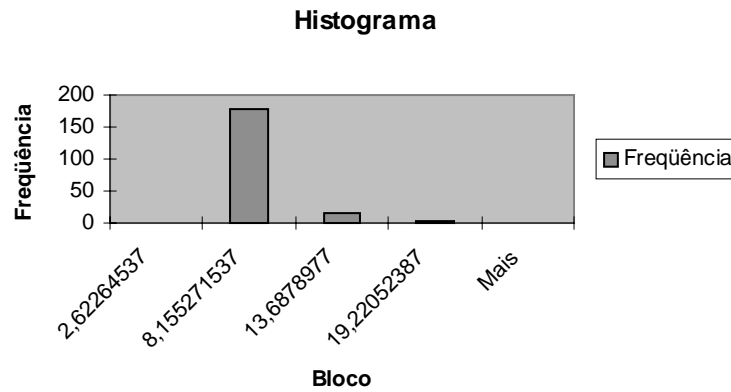
2 – the application of the Monte Carlo method in management accountancy enables the administrator to project economic scenarios for his business using the different results from simulated breakeven-points.

3 – The more repetitions there are in a simulation the greater the clarity of objectives, in the case of sales, the administrator can have.

4 -The application of the interval of confidence was fundamental to confirm that the average and standard deviation of the distribution of sales are coherent given an volume of sales of 5000 units.

The distribution of the breakeven-point is constructed in the excel program beside the histogram to follow showed, however that the distribution of the breakeven-point is not normal agreement corroborated in the test of adherence. the result does not invalidate the monte carlo method of cvp analysis under uncertain conditions defended by Jaedicke and Robichek. The study of this distribution requires a very deep knowledge of the ambit of the statistical theory.

Gráfico 1 Histogram of the simulated breakeven-point



Excel uses intervals for the creation of the histogram, coforms as demonstrated

| <i>Block</i> | <i>Frequency</i> |
|--------------|------------------|
| 2,6226453 | 1 |
| 7 | |
| 8,1552715 | 177 |
| 4 | |
| 13,687897 | 17 |
| 7 | |
| 19,220523 | 4 |
| 9 | |
| Mais | 1 |

f) In relation, the distribuiton of sales presented by Jaedicke and Roberticke presented a new graph.

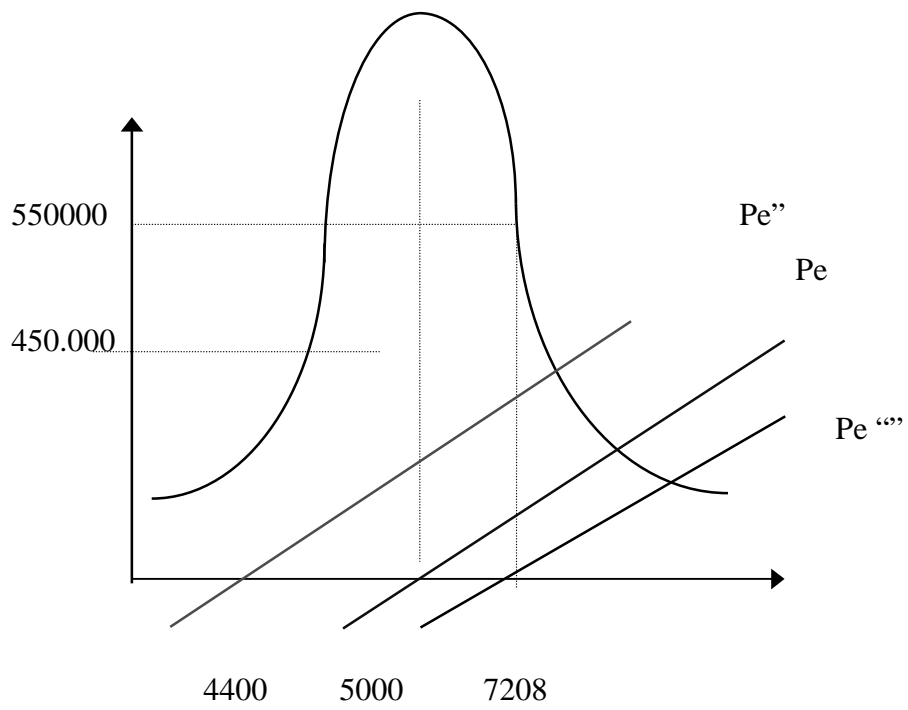


Figura – 3 Distribution of sales with the simulated breakeven-point

6 – Conclusions

a – The hypothesis, like the objectives in this study are confirmed in the results obtained by the application of the Monte Carlo Method of CVP analysis.

B – This work demonstrated that the variables involved in CVP analysis must, in principle be considered by management accounting as uncertain and only, henceforth, as an identification of risk, these must be identified with some precision.

C – The concepts presented in revision of the literature need to be investigated more dynamic vision of investment made in business.

D – The relations developed between the variables of cvp analysis can be applied to other areas of accountancy.

e- The articles written by Freitas, Corrar, Salybi and Hertz demonstrate the importance of the application of the Monte Carlo Method in informing accounts and administration.

F – Statistics allied to accountancy contribute to the advancement of information obtained by accountancy.

One following step should be to study the distribution of each variable that makes up CVP analysis so that they can be known as characteristics of this variables.

6 Referência Bibliográfica

- ¹ JAEDICKE, Robert K. and ROBICHEK, Alexander A . Cost-volume-profit analysis under conditions of uncertainty. **Accounting Review**. Florida, vol. 39, p. 917-926.Oct. 1964.
- ² FREITAS, Jr. Antônio de A. **Avaliação na análise fundamentalista usando o método de monte carlo**. Associação Brasileira de analista de Mercado de Capitais.Fortaleza.nov. 1995.
- ³ CORRAR, Luiz João **O modelo econômico da empresa em condições de incerteza: aplicação do método de simulação de monte carlo**.Cadernos de estudo. São Paulo: FIEPECAFI,n. 8, p. 1-11, abril 1993.
- ⁴ SALIBY, Eduardo. **Aplicação da amostragem descritiva na análise de risco**. Relatório . Rio de Janeiro: COPPEAD, n.185, p. 1-16, maio 1987.
- ⁵ HERTZ, David. “Risk analysis in Capital investment”, **Harvard Business Review** . v: 42, p. 96-105, Jan./fev. 1964.
- ⁵ WONNACOTT, Thomas H. & WONNACOTT **Introductory statistics for business and economics**. 4.ed. John Wiley & Sons.New York 1990. p.815.

6.1 Bibliografia Consultada.

- ABREU, Paulo F. Simas P. ; Stephan. **Análise de investimentos**. Rio de Janeiro: Campus, 1982. 280 p.
- ADAR, Z, A .BARNEA, and LEV, B. Comprehensive cost-volume-profit analysis under certainty. **The Accounting Review**, Florida, Jan.1977, p137- 49.
- BIERMAN Jr., Harold : DEBRIN Allan R. **Contabilidade gerencial**. 2. ed. Rio de Janeiro: Guanabara dois,1979. 390 p.
- BREALEY, Richard ; MYERS, STEWART, C. **Princípios de finanças empresariais**. 3 ed. [Lisbos]. McGraw-Hill de Portugal, 1988. 924 p.
- BUSBY, S.L. Extending the Aplicability of probabilistic manegement planing end control methods. **The Accounting Review**, Florida, Jan.1974, p- 42-49.
- CASTRO, Lauro Sodr  V de. **Pontos de Estatística**. Riode Janeiro: ed.cient fica. Editora cient fica. 1967. 261 p..
- CORRAR, Luiz Jo o **O modelo econ mico da empresa em condi es de incerteza**: aplica o do m todo de simula o de monte carlo.Cadernos de estudo. S o Paulo: FIEPECAFI, n. 8, p. 1-11, abril 1993.
- CHUNG, Kee H. Cost-volume-profit analysis under uncertainty when the firm has production flexibility. **Journal of BusinessFinance & Accounting**. Memphis, n 20(4). p. 583-590/ June 1993.
- COSTA, Magnus Amaral da. Rela o custo-volume-lucro para multiprodutos. **Revista Brasileira de Contabilidade**. Bras lia: CFC n. 64,p.5-9, jan./mar.1988.

De ROCHI, Carlos A . Análise cvl nas empresas de produção múltipla. **Revista Brasileira de Contabilidade**. Brasília: CFC. n. 38. p.1981.

_____ Algumas reflexões sobre a decisão “fazer ou comprar”. **Revista do Conselho Regional do Rio Grande do Sul. Porto Alegre:** n. 64. 10-13 jan./mar. 1991.

_____ Modelo estocástico para análise custo-volume-lucro. **Revista do Conselho Regional do Rio Grande do Sul.** Porto Alegre: n.39, p. 1981.

DEVINE, C.T. “Poundaries and potential of reporting on profit volume realationships.N.A A . **Bulletin** . vol 42, n. 5 , Jan.1961, p.13

FERRARA, William. Cost Management Accounting : the 21st century. **Manegement Accounting**. New Jersey, v: 77, n. 6, p. 30-36. Dec. 1995.

FERRARA,W. L, J.C. HAYYA, and D. A . NACHMAN. Normalcy of profit in the Jaedicke-Robichek Model. **The Accounting Review**, april 1972, p.299-307.

FREITAS, Jr. Antônio de A. **Avaliação na análise fundamentalista usando o método de monte carlo**. Associação Brasileira de analista de Mercado de Capitais.Fortaleza.nov. 1995.

HILTON The Determinants of Costs information value: na illustrative analysis **Journal of Accounting Research**, vol. 17, n 2 Autumn, 1979, 411-35.

HILIARD, J.E., and R. A . Leitch. Cost-Volume-Profit Analysis under uncertainty: A long Normal Approach. **The Accounting Review**. January 1975, p 69 -80.

HERTZ, David. “Risk analysis in Capital investment”, **Harvard Business Review** . v: 42, p. 96-105, Jan./fev. 1964.

IJRI,Y. & ITAMI, H. Quadratic cost-volume relationship and timing of demand information. **The Accounting Review**, vol 48 ,octuber, 1973, p. 724-37.

_____ Adaptive behavior: Manegement control and information analysis. Studies in Accounting Research,: **American Accounting Association**, n.15 .Sarasota,Florida 1977, p.156

HORNGREEN, Charles T. **Introdução à contabilidade gerencial**. 5 ed. Rio de Janeiro. Prentice Hall do Brasil. c 1985.509 p.

JAEDICKE, Robert K. and ROBICHEK, Alexander A . Cost-volume-profit analysis under conditions of uncertainty. **Accounting Review**. Florida, vol. 39, p. 917-926.Oct. 1964.

JOHNSON, G, and SIMIOK.S. Multiproduct C-V-P analysis under uncertainty. **Journal of Accounting Research**, vol 9, n. 2, Autumn, 1971, 278-86.

KABAK, Irwin W. and SIOMKOS, George J. . Adapting to a changing system procedure. **Manegerial Accounting**. p. 61-62.Apr. 1993.

LIAO,M. Model Sampling: A Stochastic cost-volume-profit analysis. **The Accounting Review**, October .1995, p. 780-90 .

MALOO, Man C. A Pratical Approach for incorporating uncertainty in the conventional cost-volume-prfit model. **Akron Business and Economic Review**. v. 22, n. 4, p . 29-40, Winter 1991.

NAKAGAWA, Massauiki. **Custeio baseado em atividade**.SãoPaulo.Atlas.1994.
p.95

PHILLIPS, Paul A . Welsh Hotel: cost-volume-profit analysis and uncertainty. **International Journal of Contemporary Hospital Manegement.** v. 6, n.3, p. 31-35, 1994.

ROSS, Stephen A .;WESTERFIELD, Randolph W.; JAFFE, Jeffrey F. **Administração financeira:** corporate finance.Atlas.1995. 697 p.

SALIBY, Eduardo. **Aplicação da amostragem descritiva na análise de risco.** Relatório . Rio de Janeiro: COPPEAD, n.185, p. 1-16, maio 1987.

SARDINHA, José Carlos. **Controladoria avançada.** Apostila. Rio de Janeiro: Case Consultores, p. 50-54.

SHERMAN, Lawrence F. SHIM, Jae K. and HARTNEY Mark. Short run break-even analysis for real estate projects. **Real Estate Issues.** p 15-20. Springer/Summer.1993.

STEVENSON, William J. **Estatística aplicada à administração.** São Paulo: Harbra.1981. 490.p.

Von GERSDORFF, Ralph C.J. **Análise do “ponto crítico no brasil”parte I:** situação, problemas e sugestões. Revista Brasileira de Contabilidade.Brasília: CFC n. 39, p.4-7 . out./dez. 1981.

YARDIN, Amaro Ramón & JAÚREGUI, Hugo A . Rodriguez. **Emprego da análisecvl nas decisões sobre a alternativa de “produzir ou comparar”.**Conselho Regional de Contabilidade do Rio Grande do Sul.Porto Alegre n.30,1981 p.2-8

_____ **Análise do “ponto crítico no brasil” parte II (final):** situação,
problemas e sugestões. Revista Brasileira de Contabilidade. Brasília: CFC n.
39.1981.p.16-24.

WONNACOTT, Thomas H. & WONNACOTT **Introductory statistics for
business and economics.** 4.ed. John Wiley & Sons. New York 1990. p.815.