TRANSITIONAL IRREGULARITY

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Abstract: All the postsocialist economies are suffering from a transition from a stable steady state or cycle into the chaotic region. Economic activity is much more erratic.
The basic objectives of this paper were: first, to establish the logistic model of the gross national product (GNP) ; second, to show how the dynamics for iterators to this model are the same as for the logistic equation, and third, to introduce the Phillips-Okun law into the model. The Phillips-Okun law was adopted. Within chaotic region the time path of real GNP is highly sensitive in variation in parameter values or initial values.
The behaviour of the gross national product is much less predictable.

K e y W o r d s: the gross national product (GNP) , chaotic behaviour, the Phillips-Okun law.

Introduction

Economies in transition experience a process of structural change. There were no previous experience with economic transformation of this magnitude. It is possible that a transition from a stable steady state or cycle into the chaotic region occured in transition economies. Many transition economies switched into a period in which economic activity is much more erratic. It is important to understand that the adjustment of the transition economies and the transition from central planning to market economy is a phenomenon of chaos. It is thus not surprising that these economies suffer from a severe transition crises.

A simple growth model which generate chaos

Nominal gross national product (or nominal GNP) represents the value, at current market prices, of all final goods and services produced within some period by nation.. On the other hand, real GNP expresses nominal GNP corrected for inflation.

Potential GNP or "high employment output" represents the maximum level of GNP that can be sustained with a given state of technology, population size, inflation; sometimes called "the level of output corresponding to the natural rate of unemployment".

The difference or gap between potential GNP and realGNP is defined as GNP gap. In this sense, business cycles are generated because the gap between potential and actual GNP widens and shrinks. For example, if the economy attempts to produce more than its potential output, then prices will begin to rise. If the economy produces less than its potential, then unemployment will begin to rise. Depending on the relationship between the level of actual and potential GNP economy has alternated between boom and recession. A large GNP gap means economic instability.

Economic growth is usually measured as the annual rate of increase in a nation's real GNP. Namely, if the growth model assumes that the growth rate of GNP, λ , is constant, then

$$\frac{X_{n+1} - X_n}{X_n} = \lambda \qquad (1)$$

for some number λ independent of n. X denotes real GNP.

The growth law is

$$X_{n+1} = X_n + \lambda X_n = (1 + \lambda) X_n$$

or

$$X_n = (1 + \lambda)^n X_0 \qquad (2)$$

where X_0 is the initial real GNP with which we start our observation at time 0. In other words, knowing λ and measuring X_0 would suffice to predict X_n for any point in time. The most simple growth model would assume a constant growth rate of real GNP, but in that situation we find unlimited growth which is not realistic.

We will assume that actual real GNP is restricted by a potential GNP, but this premise requires a modification of the growth law. Now the growth rate depends on the actual size of real GNP, X, relative to its potential size X_p. We introduce x as $x=X/X_p$. Thus x ranges between 0 and 1, i.e., we can interpret x=0.85 , for example, as the actual size of real GNP, X, being 85% of its potential size , X_p.

Again, we index x by n, i.e. write x_n to refer to the size at ime steps n = 0,1,2,3,... Now growth rate of real GNP is measured by the quantity

$$\frac{x_{n+1} - x_n}{x_n} \quad \text{"}\textit{direct proportional to}\text{"} \quad 1 - x_n$$

or, after introducing a suitable constant δ

$$\frac{x_{n+1} - x_n}{x_n} = \frac{1}{\delta} \quad (1 - x_n)$$

Solving this last equation yields the growth model

$$x_{n+1} = x_n + \frac{1}{\delta} x_n (1 - x_n) \qquad (3)$$

This model given by equation (3) is called the logistic model. For most choices of δ , there is no explicit solution for (3). Namely, knowing δ and measuring y_0 would not suffice to predict y_n for any point in time, as was previously polssible. It is observed that continued iteration requires higher and higher computation accuracy if we insist on exact results. In this sense, computed predictions in our model can be totally wrong.

This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

It is showed that iteretaion process for the logistic equation , $y_{n+1} = \mu\, y_n (1 - y_n)$ is equilvalent to the iteration of our growth model (3). Namely, our growth model (3) is the same as

$$y_{n+1} = \mu\, y_n (1 - y_n) \qquad (4)$$

when using the identification

$$y_n = \frac{1}{1+\delta} x_n \qquad \textit{and} \qquad \mu = \frac{1}{\delta} + 1 \qquad (5)$$

We compute y_{n+1} using (5) and the logistic iteration then check if the result agree with the iteration using (4). We obtain

$$y_{n+1} = \frac{1}{1+\delta} x_{n+1} = \frac{1}{1+\delta}\left[x_n + \frac{1}{\delta} x_n (1 - x_n) \right]$$

$$= \frac{1}{\delta} x_n - \frac{1}{\delta\,(\delta+1)} x_n^{\,2}$$

and on the other hand

$$y_{n+1} = \mu\; y_n (1 - y_n) = \left(\frac{1}{\delta} + 1 \right) \frac{1}{1+\delta}\, x_n \left(1 - \frac{1}{1+\delta}\, x_n \right)$$

$$= \frac{1}{\delta} x_n - \frac{1}{\delta\,(\delta+1)} x_n^{\,2}$$

We can conclude that the logistic equation (4) of the proposed growth model is equivalent to the quadratic iterator (3) using

$$y_n = \frac{1}{1+\delta} x_n \qquad \qquad \textit{and} \qquad \qquad \mu = \frac{1}{\delta} + 1.$$

It is important because the dynamic properties of the logistic equation (4) have been widely analyzed (Li and Yorke (1975), May (1976)). It is obtained that

(i) For parameter values $0< \mu <1$ all solution will converge to $y = 0$;
(ii) For $1 < \mu <3.57$ there wcist fixed points the number of which depends on μ ;
(iii) For $3.57 < \mu < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period.

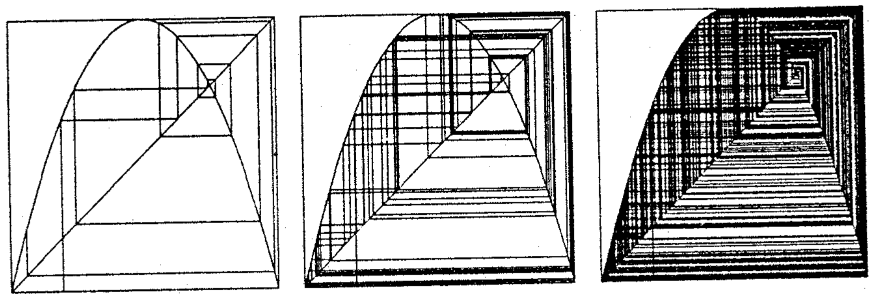

Figure 1. Iteration of the logistic map for a chaotic state at $\mu = 4$ (*Source : Peitgen HO, Jürgens H and D Saupe, pg. 59 , 1992)*

Okun (1962) established a relationship between the extent to which actual output is depressed from its potential level by unemployment in excess of its natural rate. The latter he assumed to be 4 percent. Okun ended up with the following relationship between output, potential output, and unemployment:

$$X_p = X + 0.032\, X\, (u - u^p) \qquad (6)$$

with u as the unemployment rate in percent, X_p as the potential output, X as the actual output, $\alpha = 0.032$ and $u^p = 0.04$. When unemployment takes on its natural rate of 4 percent, the actual and potential output coincide. In other words, one percentage point more in unemployment means 3.2 percent less in GNP.

This relationship between unemployment and the loss in GNP has become famous as Okun's law.

Record highs in real GNP and income do not necessarily indicate a satisfactory performance of the economy; it might be the case that for lower unemployment the economy would have done even better.

As long as the difference between potential and actual output exists, the economy exihibits underutilised capacities. As long as the output gap exists, additional economic actions must be undertaken until it finally disappears.

On the other hand, Phillips (1958) set out empirical evidence to support the view that there was a significant realation between the percentage change of money wages and the level of unemployment - the lower unemployment, the higher the rate of change of wage. This relationship, which became known as the Phillips curve, has attracted considerable analysis. Its main implication is that, since a particular level of unemployment in the economy will imply a particular rate of wage increase, the aims of low unemployment and a low rate of inflation may be inconsistent. The government must choose between the feasible combinations of unemployment and inflation, as shown by the estimated Phillips curve. However, the relation between unemployment and inflation has not been sufficiently stable in practice to permit ecact judgements to be made.

$$(\pi - \pi^{e}) = - \beta (u - u^{p}) \qquad (7)$$

And , finally Phillips-Okun's law

$$X_{p} = X - \frac{\alpha}{\beta} X \left(\pi - \pi^{e} \right) \qquad (8)$$

i.e.

$$X_{p} = X - \frac{\alpha \gamma}{\beta} X \qquad (9)$$

where $(\pi - \pi^{e}) = \gamma$

When (9) is divided by X_p

$$1 - x_{n} = - \frac{\alpha \gamma}{\beta} x_{n} \qquad (10)$$

where

$$\frac{X}{X_{p}} = x_{n}$$

(10) in (3)

$$x_{n+1} = x_n - \frac{\alpha \gamma}{\beta \delta} x_n^2 \qquad (11)$$

This upside-down parabola cuts the abscissa at 0 and

$$x_{n2} = \frac{\beta \delta}{\alpha \gamma} \quad ; \qquad at$$

$$x_{n1} = \frac{\beta \delta}{2 \alpha \gamma} \qquad (12)$$

it has its maximum value of

$$x_{(n+1),\max} = \frac{\beta \delta}{4 \alpha \gamma}$$

It is of importance whether the solution of (11) can display chaotic behaviour or not. If the criterion (13) of Li and Yorke (1975) any difference equation $x_{n+1}=f(x_n)$ mapping an interval J into itself will show up chaotic behaviour if there exists a point $x^* \in J$ with

(13) $f^3(x^*) \leq x^* < f(x^*) < f^2(x^*)$,

with $f^k(x)$ the k-th iterate of f, i.e. $f^1(x) = f(x)$,
$f^k(x)=f(f^{k-1}(x))$, k=2,3,...

Thus any "one-humped" difference equation fulfilling condition (13) will posses a chaotic region.

Two properties of the chaotic solution are important for economic theory. First, given a starting point $x_{(0)}$ the solution is highly sensitive to variations of the parameter a; second, given the parameter a the solution is highly sensitive to variations of the initial point $x_{(0)}$. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a completely different, i.e. chaotic manner.

To prove that (13) is met we will proceed sismilar to Day (1982). THe interval that (11) has to map into itself is [0, x_{n2}]. Hence, $x_{n+1, max}$ must be smaller than or equal to x_{n2}.

Than

$$\frac{\beta\,\delta}{4\,\alpha\,\gamma} < \frac{\beta\,\delta}{\alpha\,\gamma}$$

or

$$(\pi - \pi^{e}) < \frac{\beta\,\delta}{\alpha} \qquad (14)$$

must hold. For any parameter values of α , β , γ and δ fulfilling this equation, the graph of (11) has the shape of Figure 2.

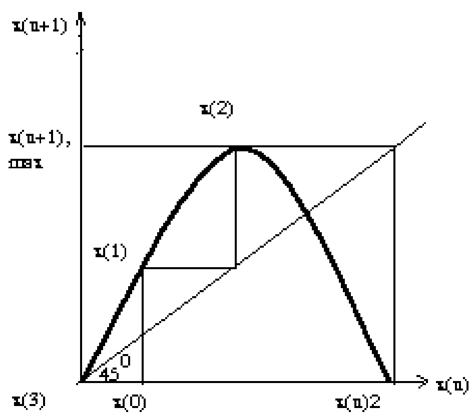

Figure 2.

The initial value x_0 is chosen in such a way that $x_{(2)} = x_{(n+1), max}$, hence $x_{(3)}$ must be zero.Taken together we have:

$x_{(3)} < x_{(0)} < x_{(1)} < x_{(2)}$

so that the theorem (13) of Li and Yorke is met.

Thus the solution of the modified growth rate model must behave chaotically for particular parameter values.

Conclusion

Surely, this chaotic movement of x_n cannot be interpreted as a pure business cycle-type behaviour. The aperiodic solutions in the chaos region of (11) can ab best be interpreted as a random walk, although they are completely deterministic. Yet business cycles display much more regularities. In this sense, they cannot be interpreted as a random walk. So at best business cycles can be partly related to chaotic behaviour of deterministic process.

By some slight modifications the standard growth rate model will lead to such chaotic behaviour of the endogenous variable. Within the chaotic region the time path of this variable is highly sensitive to variations in parameter values of initial values. Hence long term predictions based on empirical observations can be extremly difficult as long as the endogenous variable will fall within the chaotic region.

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